I. A Cosmological Primer

Timetable

Monday	Tuesday	Wednesday	Thursday	Friday
	4pm		4pm	4pm
	4pm		no lecture	4pm

- Can we have lectures from 16.00-16.50?
- Timetable shows problems class, but will hold drop-in session in my office
  - would you prefer a set 2-hour block, or an open door?
- Lecture videos on blackboard

About Me

- Dr Ben Maughan
  - Degree: Cardiff
  - Ph.D: Birmingham
  - Chandra Fellow: Harvard-Smithsonian Centre for Astrophysics
- Research:
  - X-ray properties of galaxy clusters
  - Cosmology
- Room 4.19
- ben.maughan@bristol.ac.uk
- http://www.star.bris.ac.uk/bjm/interesting

Course Overview

Observations of galaxy clusters (primarily in X-rays) for cosmological tests

- ★ Cosmological primer
  - incomplete overview of parts of cosmology important for cluster studies
- ★ Galaxy clusters and their observation
  - mass determinations
- ★ Cluster baryon fraction tests
  - constraints on matter density and standard bucket
- ★ Cluster mass function
  - growth of structure and volume tests

Assessment

- 2,000 – 3,000 word article on Cosmology with Galaxy Clusters
  - 2,000 plenty!
- include figures/equations/references as appropriate
  - references should give enough info for reader to locate source
- aim at level M student level of knowledge
- write in popular science style – readable & engaging
  - see e.g. Physics World articles
- don't need to cover whole content of lectures
  - decide what to include to make interesting and informative article

- reader should appreciate how galaxy clusters can be used to obtain cosmological constraints

Assessment

- deadline – 23.59 on 18 Nov 2011
  - submit via blackboard
- style and clarity (40%)
  - writing style, grammar, presentation & scope of material, level of material, structure and flow
- scientific content (60%)
  - quality and quantity of technical content, analytical & critical ability, quality of conclusions, use of literature (beyond references in lectures), understanding of material

Cosmological Primer

- Cosmological models
- Distances
- Volumes
- Dark Energy
- Growth of Structure

Cosmological Models

In simplest models of expanding Universe, only the gravitational force of matter influences dynamics of expansion.

Three possible scenarios:

- ★ **open** – not enough matter to halt expansion
- ★ **closed** – more than enough matter to halt and re-collapse
- ★ **flat** – boundary case

The mass-energy density required for flat Universe is referred to as the **critical density** $\boldsymbol{\rho_c}$

- ★ with cosmological constant, continual accelerated expansion possible in closed Universe

Cosmological Models

In general relativity, these cases are thought of in terms of curvature of spacetime,

- ⋆ open – negative curvature
- ⋆ closed – positive curvature
- ⋆ flat – zero curvature (Euclidean geometry)

See level 7 GR course for details

We will only consider flat cosmologies!

Density Contributions

It is convenient to write the density of matter in the Universe as a ratio to the critical density:

$$\Omega_M = \frac{\rho_M}{\rho_c}$$

Radiation also contributes energy density to Universe, which we can also write as

$$\Omega_R = \frac{\rho_R}{\rho_c}$$

Ω_R dominated in early Universe but is negligible now as drops off faster with z than matter

- ⋆ both decrease as 1/V but radiation also redshifted

Density Contributions

In GR, Einstein proposed a **cosmological constant (Λ)** term in equations to allow static Universe solutions

Inflation theory and observations (e.g. CMB) all suggest **flat Universe**, while other observations suggest **$\Omega_M < 1$**

★ new interest in Λ to provide extra energy density

Λ acts as constant (in space and time) energy density, and we can write

$$\Omega_\Lambda = \frac{\Lambda}{\rho_c}$$

N.B. Λ is constant, but Ω_Λ changes with time

Density Contributions

We can write to total contribution of the density terms:

$$\Omega_{tot} = \Omega_M + \Omega_R + \Omega_\Lambda$$

and flatness requires $\Omega_{tot} = 1$

We will assume flat Universe throughout, and neglect Ω_R

Distances

There are several different ways of measuring distances in cosmology

- ★ differ due to expansion of Universe

Proper distance d to an object is distance measured if we froze Universe and stretched tape measure to object

- ★ In expanding Universe, d increases with time

Useful to define **comoving distance χ** which includes expansion

- ★ objects moving apart due only to expansion have constant χ

Distances

Proper distance and comoving distance related by

$$d(t) = a(t)\chi$$

where a(t) is the **scale factor**

★ describes scale of Universe as function of time

Distances – Hubble's Law

Proper distance and comoving distance related by

$$d(t) = a(t)\chi$$

where a(t) is the **scale factor**

⋆ describes scale of Universe as function of time

Differentiate this expression wrt time to get Hubble's law:

$$v(t) = \dot{a}(t)\chi$$

$$v(t) = \frac{\dot{a}(t)}{a(t)} d(t) \quad \text{where} \quad \dot{a} = \frac{da}{dt}$$

Distances – Hubble's Law

$$v(t) = \frac{\dot{a}(t)}{a(t)} d(t)$$

$$v(t) = H(t) d(t)$$

$\dot{a}/a$ is the Hubble parameter H(t) or H(z)

- ⋆ expansion rate per unit size

Often written as

$$H(z) = H_0 E(z)$$

where E(z) is a function describing the evolution of H.

For a flat Universe with a cosmological constant

$$E^2(z) = \Omega_M (1+z)^3 + \Omega_\Lambda$$

Distances - Redshift

Consider photon emitted at t_e and received at t_0

- ★ Universe expanded by factor $a_0/a(t_e)$
- ★ a_0 is $a(t_0)$ – N.B. some texts normalise a so $a_0=1$
- ★ Wavelength of photon stretched by same factor:

$$1 + z = \lambda_0/\lambda_e = a_0/a(t_e)$$

$$\frac{a}{a_0} = \frac{1}{1+z}$$

Distance – Redshift Relations

Derive expression for distance to object with redshift z

- ★ depends on cosmological model
- ★ observational data that provide d and z used to constrain cosmological models

Consider photon emitted at t_e and received at t_0

In time *dt*, photon travels proper distance *cdt*, or comoving distance

$$d\chi = \frac{c}{a(t)}dt$$

The total comoving distance to the source is then

$$\chi = \int_{t_e}^{t_0} \frac{c}{a(t)}dt$$

Distance – Redshift Relations

$$\chi = \int_{t_e}^{t_0} \frac{c}{a(t)} dt$$

can be rewritten as

$$\chi = c \int_{t_e}^{t_0} \frac{1}{\dot{a}} \frac{da}{a} \qquad \text{where} \qquad \dot{a} = \frac{da}{dt}$$

From the relation between *a* and *z* we can write

$$da = -a_0 (1+z)^{-2} dz$$

which we substitute in above and rearrange to give

$$\chi = \frac{c}{a_0} \int_0^{z_e} \frac{a}{\dot{a}} dz$$

Distance – Redshift Relations

recall $\dot{a}/a = H_0 E(z)$ so can write comoving distance as:

$$\chi = \frac{c}{a_0}\int_0^{z_e} \frac{a}{\dot{a}} dz = \frac{c}{a_0 H_0}\int_0^{z} \frac{dz}{E(z)}$$

and the proper distance to z at the present time is just

$$d(t_0) = a_0 \chi$$

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If know cosmological parameters in E(z)

★ calculate distance to object with measured z

If know distance and redshift of an objects

★ calculate cosmological parameters in E(z)

Luminosity Distance

Proper distance not measurable, but distances can be determined if physical size or luminosity of object known

For an object of known luminosity L, in flat **non-expanding** Universe, flux received is

$$F = \frac{L}{4\pi d^2}$$

where d is distance to source

- ★ equal to **proper or comoving** distance in non expanding Universe

Luminosity Distance

In **expanding** Universe, flux spread over sphere radius d_0

★ proper distance to source at present time $(d_0 = a_0 \chi)$

measured flux is reduced:

★ energy of each photon decreased by factor (1+z) due to redshift

★ time between emission of photons is increased by (1+z) due to recession velocity

Both effects together reduce flux by factor $(1+z)^2$

$$F = \frac{L}{4\pi(1+z)^2 d_0^2}$$

Luminosity Distance

$$F = \frac{L}{4\pi(1+z)^2 d_0^2}$$

Motivating us to define luminosity distance d_L such that

$$F = \frac{L}{4\pi d_L^2}$$

Thus

$$d_L = (1+z)d_0 = \left(\frac{L}{4\pi F}\right)^{1/2}$$

Angular Diameter Distance

If physical diameter D of object is known, then in flat, **non-expanding** Universe

$$d = D/\theta$$

Here d is proper or comoving distance

In **expanding** Universe, the D/θ tells us proper distance when photons were emitted

⋆ in time light from z took to reach us, Universe is factor

$$a_0/a(z) \equiv (1+z)$$

larger, so D/θ underestimates d_0 by factor (1+z)

Angular Diameter Distance

Define angular diameter distance as

$$d_A = D/\theta$$

and thus

$$d_A = D/\theta = d_0/(1+z)$$

Angular Diameter Distance

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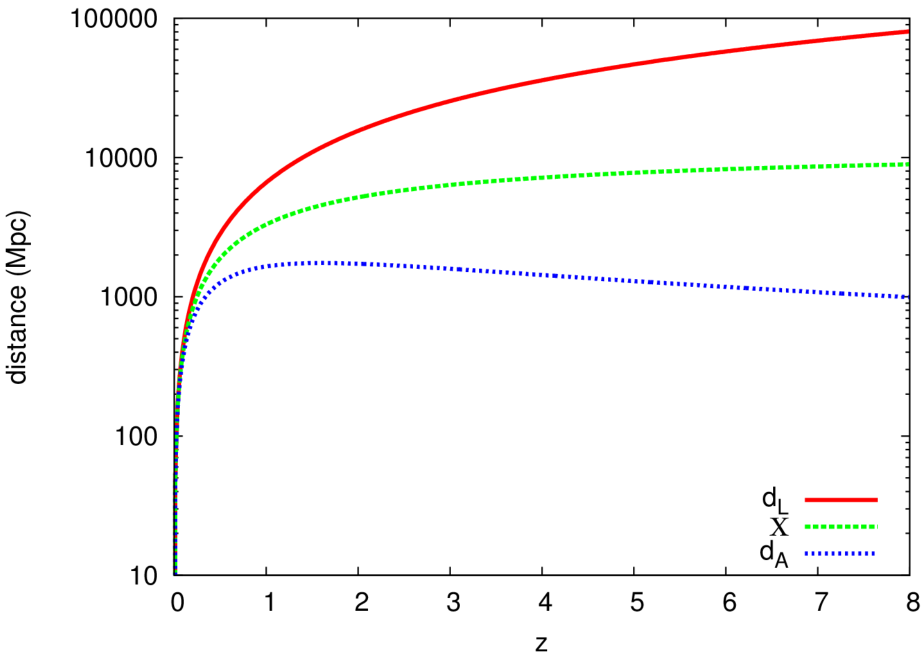


Summary of Distances

Proper distance d

★ distance measured with a tape measure at fixed time

Comoving distance

★ distance that scales with expansion

Luminosity distance d_L

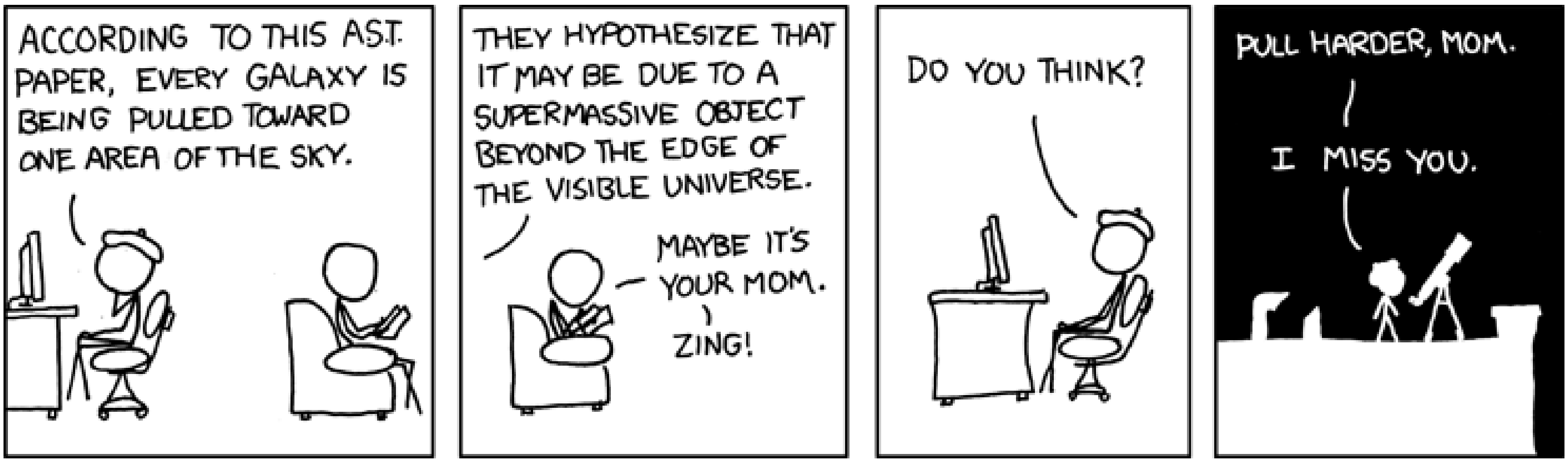
★ distance you derive from measuring flux of known L

Angular diameter distance d_A

★ distance you derive from measuring angle of known D

$$d_0 = a_0 \chi = (1+z) d_A = d_L / (1+z)$$

XKCD Break



www.xkcd.com

Comoving Volume Element

If number density of objects known from theory, can count objects in redshift range and determine volume

- ⋆ volume depends on expansion history like distances
- ⋆ constrain cosmology from volume

Comoving Volume Element

If number density of objects known from theory, can count objects in redshift range and determine volume

- ★ volume depends on expansion history like distances
- ★ constrain cosmology from volume

Consider a proper volume element of Universe with proper area dA and proper depth dr

$$dV = dA dr$$

$$dV = d_A^2 d\Omega dr$$

where $d\Omega$ is solid angle, and d_A is angular diameter distance to element

Comoving Volume Element

$$dV = d_A^2 d\Omega dr$$

write as comoving volume element dV_χ by recalling

$$\chi = (1+z)d_A \qquad d\chi = \frac{c}{a_0 H_0}\frac{dz}{E(z)}$$

to give

$$dV_\chi(z) = \frac{c}{a_0 H_0}\frac{(1+z)^2 d_A^2}{E(z)} d\Omega dz$$

which can be integrated over redshift range and solid angle to give comoving volume

Comoving Volume Element

$$dV_\chi(z) = \frac{c}{a_0 H_0} \frac{(1+z)^2 d_A^2}{E(z)} d\Omega dz$$

Thus if comoving number density of objects known:

- ★ observe number in some z range and solid angle
- ★ derive volume
- ★ constrain cosmological parameters via E(z)

Dark Energy

Combination of evidence for flat Universe and Ω_M<1 support need for cosmological constant Λ

Λ appears in Einstein's equations as source of gravitational repulsion to balance gravitational attraction of matter.

Most generally, this unknown source of repulsive gravity is referred to as **dark energy**

- ★ Λ is simplest form (energy density constant in space and time)
- ★ other models are possible

Dark Energy

Dark energy can be thought of as fluid with equation of state

$$p_\Lambda = w\rho_\Lambda$$

where w is equation of state parameter of dark energy

For cosmological constant models, w=-1

- ★ fluid with negative pressure – c.f. tension in spring
- ★ in GR -ve pressure has repulsive gravitational effect
  - not same as pressure inflating balloon (this due to difference in pressure between regions)

Dark Energy

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- ★ fluid with negative pressure – c.f. tension in spring
- ★ in GR -ve pressure has repulsive gravitational effect
  - not same as pressure inflating balloon (this due to difference in pressure between regions)
- ★ expansion of Universe decreases density but does work on fluid (opposite to e.g. ideal gas)
- ★ keeps energy density constant with time
- ★ ρ_Λ slows expansion but -ve p_Λ more than compensates

Dark Energy

Other values of w correspond to different dark energy models

- ⋆ different behaviour with time
- ⋆ more general expression for E(z) for w≠-1

$$E^2(z) = \Omega_M(1+z)^3 + \Omega_\Lambda(1+z)^{3+3w}$$

Still more general models exist in which w varies with z

By using observations to constrain the expansion history of the Universe through E(z), values of Ω_Λ and w can be determined to distinguish dark energy models

Growth of Structure

In addition to geometrical tests (distances and volumes), can constrain cosmology by growth of structure

Tiny density perturbations in early Universe grow to form observed large scale structure

★ growth sensitive to cosmology

Model density variation in terms of density contrast δ

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

where ρ is density in a region and $\bar{\rho}$ is mean matter density

★ N.B. δ depends on size of region considered

Growth of Structure

- ★ Regions with $\delta>0$ are overdense and tend to collapse
- ★ Regions with $\delta<0$ are underdense and tend to grow less dense

In non-expanding Universe, overdense regions collapse exponentially

In expanding Universe, collapse must compete with expansion

- ★ regions denser than a critical value will collapse
- ★ analogous to Universe as a whole

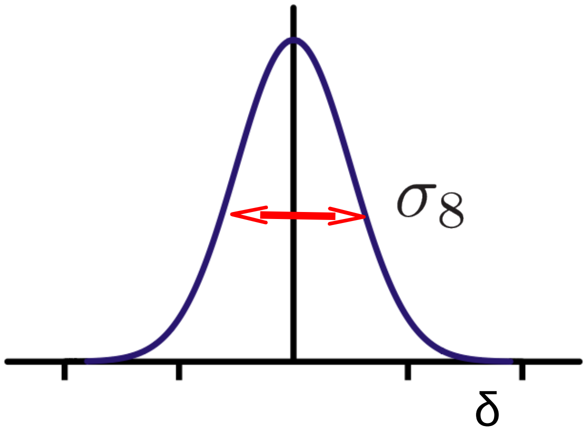
Growth of Structure

Growth of structure thus sensitive to

- ⋆ initial density distribution
- ⋆ expansion history of Universe – i.e. E(z)

Galaxy clusters sensitive to amplitude of δ distribution

- ⋆ measured via σ_8
- ⋆ sd of δ values measured in spheres of radius 8 Mpc

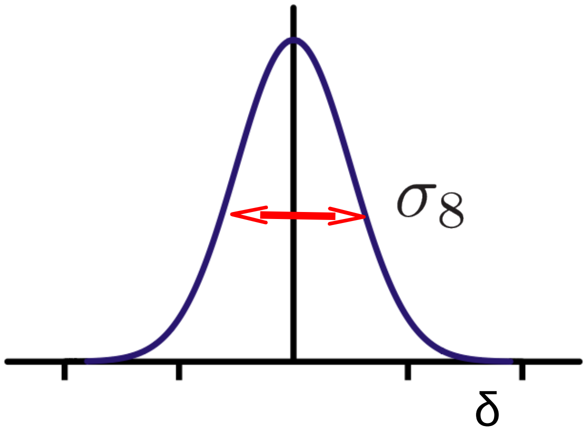


Growth of Structure

Larger values of σ_8 correspond to less uniform initial density distribution

★ more structure in Universe

Clusters grow from high δ tail of distribution so number of clusters very sensitive to σ_8



Summary

- ★ Density contributions of mass and dark energy expressed in terms of ρ_c
- ★ Several distance measures in expanding Universe
  - observable distances d_A, d_L related to z via E(z)
  - derive E(z) and constrain Ω_M, Ω_Λ by measuring d
- ★ Volume also sensitive to E(z) – constraints from known number densities
- ★ Dark Energy thought of as fluid with $p_\Lambda = w\rho_\Lambda$
  - can constrain w by measuring E(z)
- ★ Growth of structure depends on cosmological parameters
  - through competition between collapse and expansion (E(z))
  - clusters sensitive to σ_8 - variance of initial density distribution

Bibliography

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