III. Observing Clusters 2 - Scaling Relations and Self-Similarity

Observing Clusters - Summary

Galaxy clusters consists of

- Dark matter (~80%), hot gas (~15%), galaxies (~5%)

Galaxy cluster studies important for

- Measuring cluster masses for cosmology
- Investigating physical processes in clusters

Study and measure M with different techniques

- galaxies – richness, velocity dispersion
- lensing – strong and weak lensing
- SZ – probes gas properties, z independent
- simulations – verify against observations
- X-ray – spectrum of gas (ICM) gives kT

Summary of X-ray Properties

- X-ray observations of galaxy clusters allow us to measure these key properties:
  - X-ray luminosity (from images or spectra)
  - kT of the ICM (from spectra)
  - Metal abundances in ICM (from spectra)
  - Density of ICM (from surface brightness profile)
- Combining radial profiles of kT and ρ of ICM we can infer total mass assuming hydrostatic equilibrium

Observing clusters 2

* Self similarity with M and z
* Scaling relations

Self Similarity

When we describe galaxy clusters as “self-similar” we mean that clusters are simply scaled up and down versions of each other

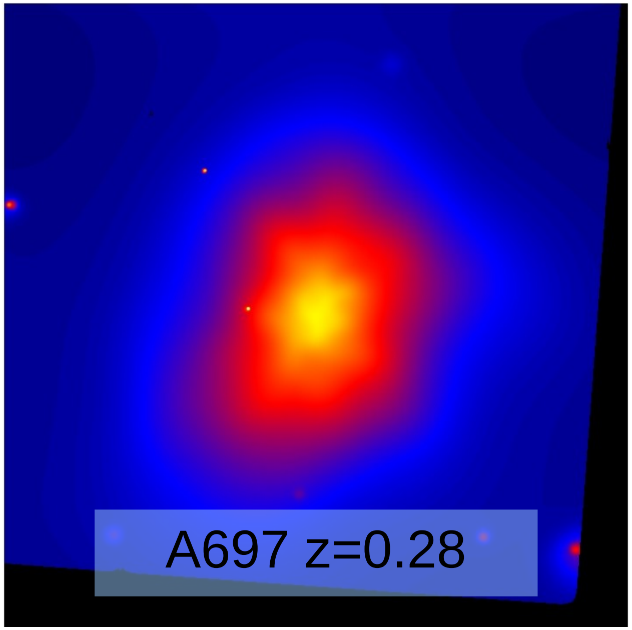
- ★ Can think of clusters being self-similar w.r.t mass or redshift
- ★ **c.f. fractals**

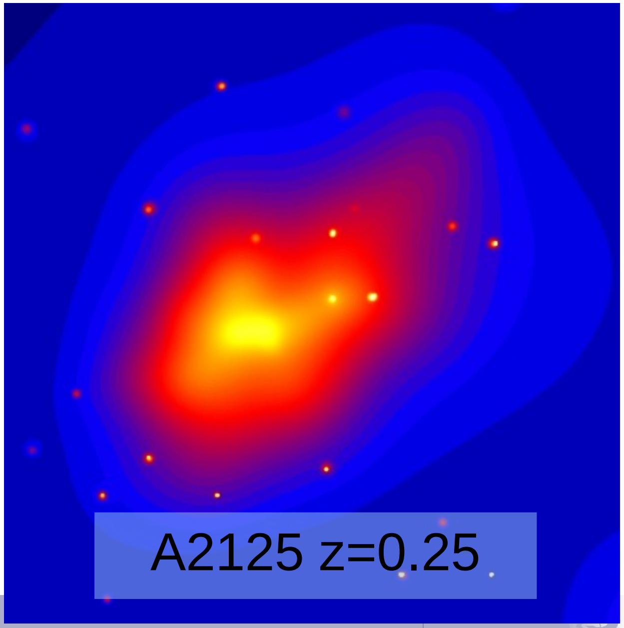
Strong Self-Similarity

One of these galaxy clusters is 10 times more massive than the other

- ★ (The images have been scaled to the same size)

Q: Which is the most massive?



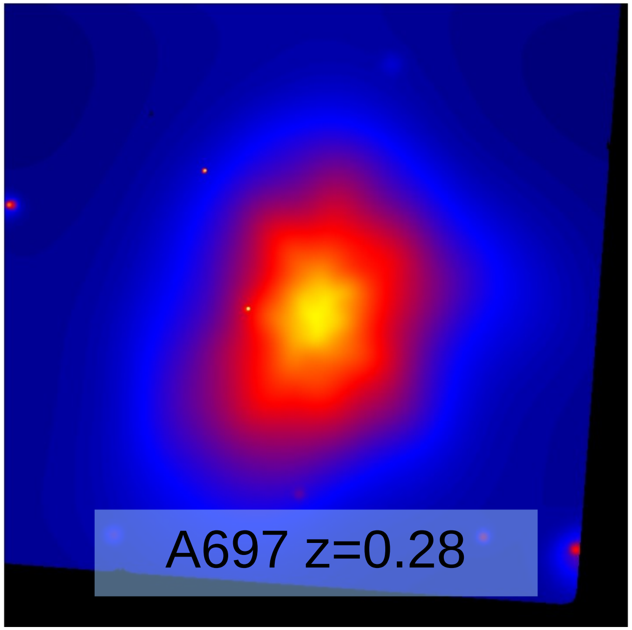


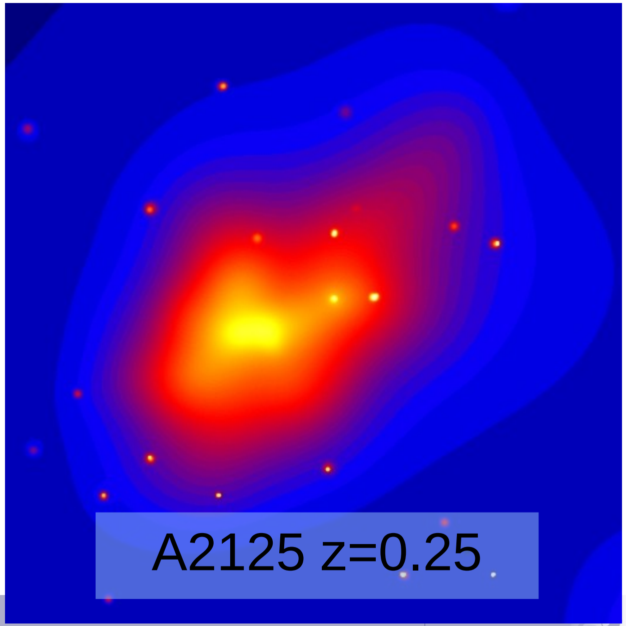
Strong Self-Similarity

Q: Which is the most massive?

A: A697, but we can't tell that from these images

Strong self-similarity means clusters of different masses are identical, scaled versions of each other

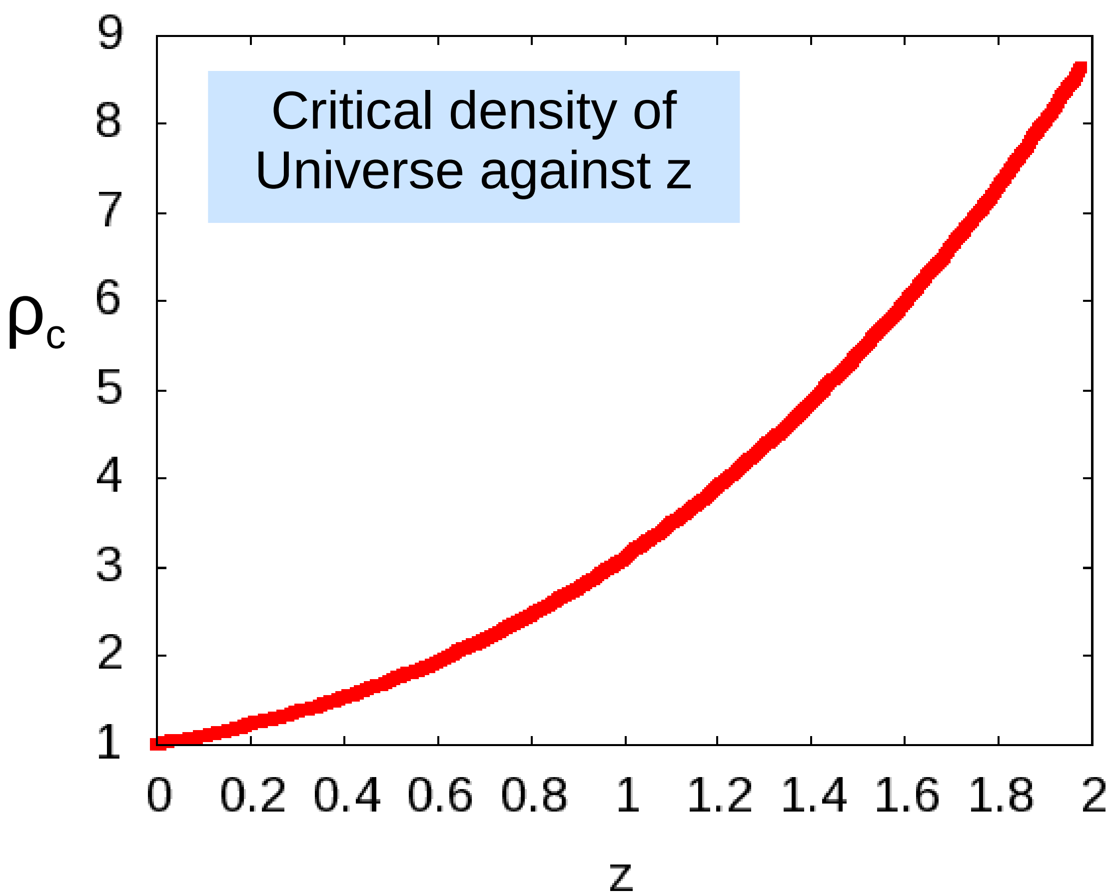




Weak Self-Similarity

Galaxy clusters are observed at z>1

- ★ At distant redshifts, we are observing a younger Universe
  - Density was higher



Weak self similarity means that as long as we account for the changing density of the Universe, a cluster at high-z is identical to a cluster of the same mass at low-z

- ★ **Self-similar evolution**

Self-Similarity

Self-similarity means all galaxy clusters essentially identical

- ★ Massive clusters are scaled up versions of less massive clusters
- ★ Distant clusters are identical to local clusters if we include factor for increasing density of Universe with redshift

Key Assumptions

The self similar model is based on the simplifying assumptions that:

- ⋆ Clusters form via a single gravitational collapse at z_{obs}
- ⋆ The only source of energy input into ICM is gravitational

N.B. Neither of these are true!

With these assumptions we can predict simple **power law** relationships between the different properties of galaxy clusters

- ⋆ **Scaling relations**

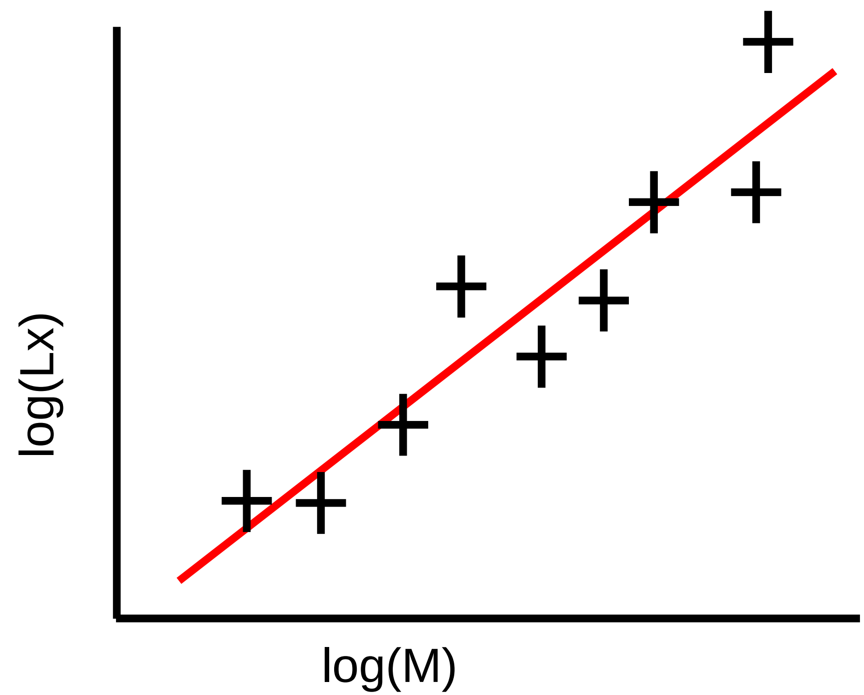
Scaling Relations

Scaling relations are **power law** relations between galaxy cluster properties (typically X-ray) such as Lx, kT, M_{gas}, M_{tot} etc.

★ e.g. The luminosity-mass (**LM**) relation describes the relationship between the X-ray luminosity and cluster mass

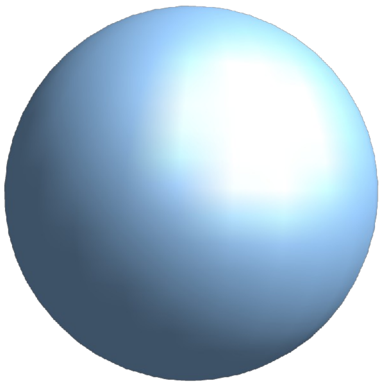
★ Measure L easily for large samples of clusters
  - estimate M
  - do cosmology!

★ Depends on accuracy and precision of scaling relation



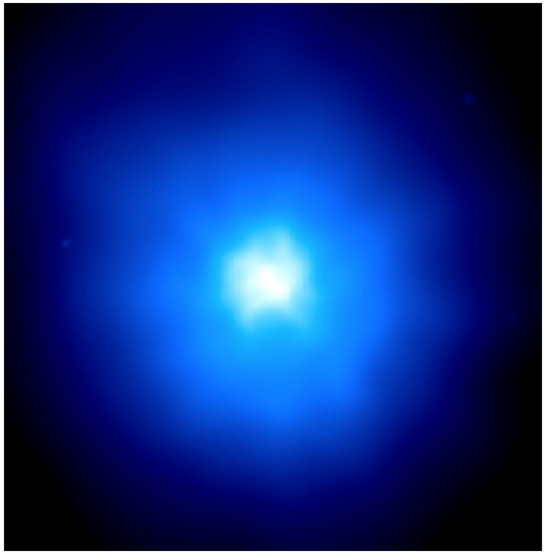
The Edge of a Cluster

When we talk about cluster properties we need to specify what radius we measure them within



The mass of this sphere is easily defined as it has a clear surface/edge

Where is the edge of this galaxy cluster?



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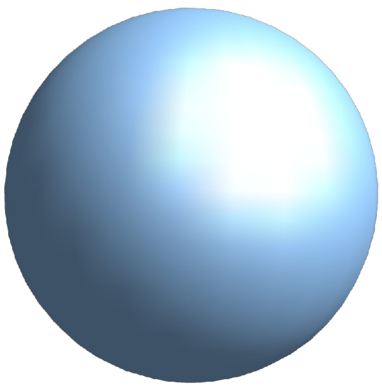

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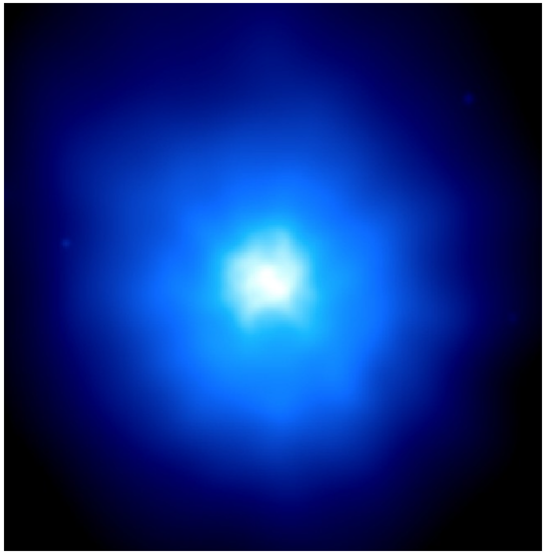
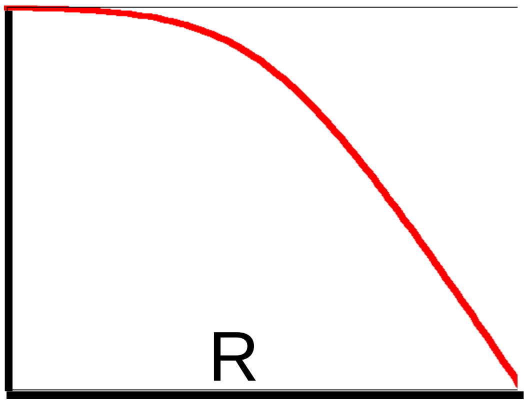


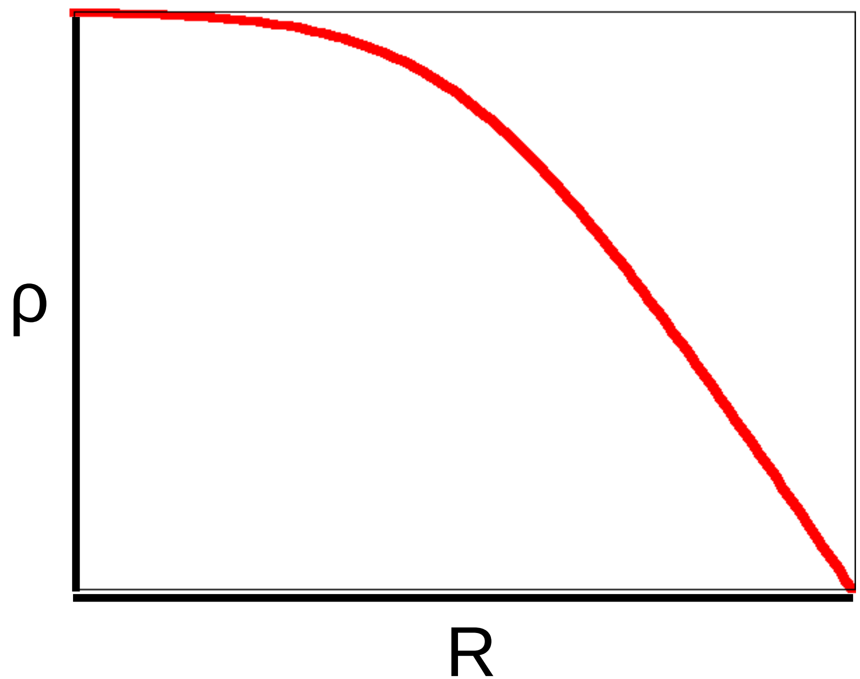


Surface brightness & gas density asymptote to zero at infinite radius (N.B. log plot)

Overdensity Radii

Use **overdensity radii** to define region in which properties are measured

- ★ A radius within which the mean density is Δ times the critical density (ρ_c) at the cluster's redshift
- ★ Clusters are centrally concentrated so larger Δ correspond to smaller radii
- ★ Write radii as R_Δ
  - e.g. R_{200} means Δ=200



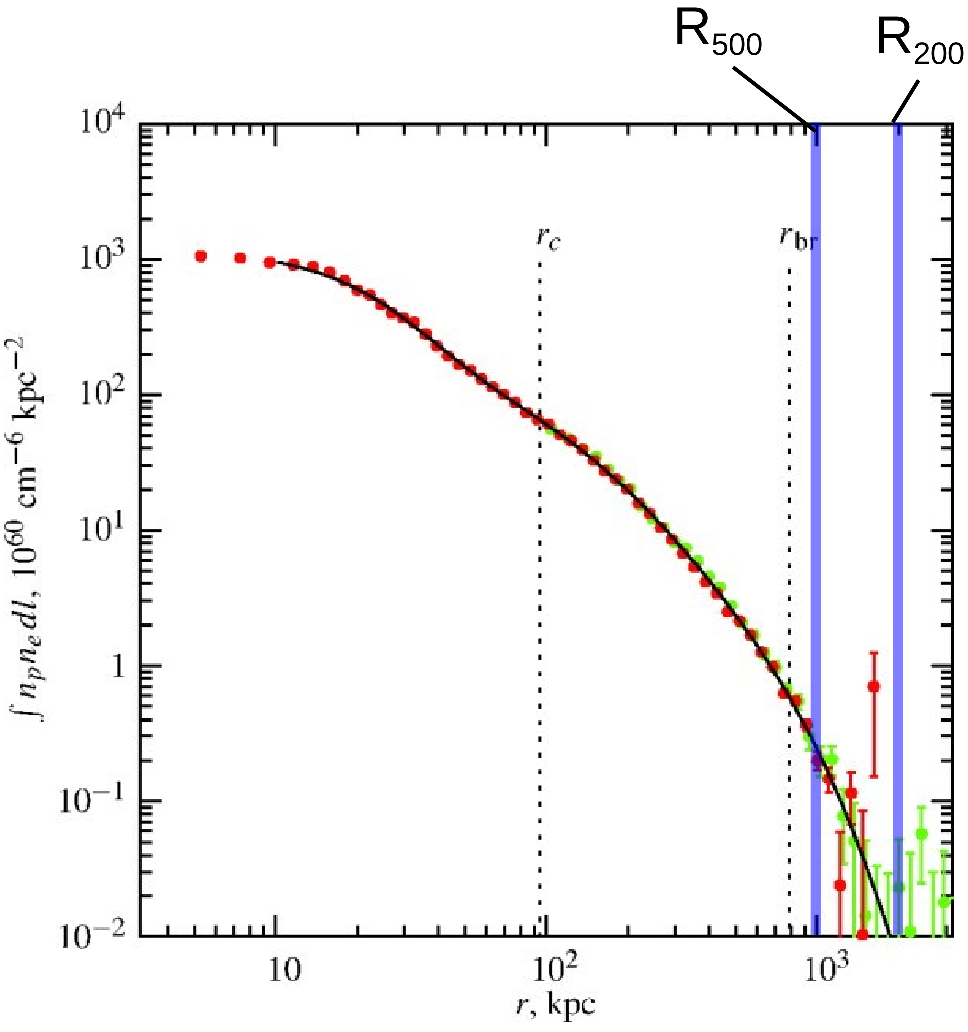
N.B. here ρ is the total mass density (not just gas)

Overdensity radii allow fair comparison of properties of clusters of different sizes, key part of self-similar model

Overdensity Radii

Simulations show that Δ=200 corresponds to **virial radius**

- ★ Radius separating relaxed part of cluster from infalling material
- ★ ≈2 Mpc (massive cluster)
- ★ R_{500} (~$0.5R_{200}$) is radius measured out to in typical X-ray observations



MT Relation

- If a galaxy cluster is **dynamically relaxed** (no recent mergers) we expect the gas and galaxies to be **virialised**:

$$2\mathrm{K} = -U$$

where K is kinetic energy and U is potential energy

- For monatomic gas with temperature T, the average kinetic energy per particle is

$$\langle K_i \rangle = \frac{3}{2} kT$$

- and total KE of gas, K, is N<K_i> where N is number of particles, so

$$K \propto NkT \propto M_{gas,\Delta} kT$$

MT Relation

- For self-similar clusters, $M_{gas,\Delta} \propto M_\Delta$, the total mass within R_Δ, so

$$K \propto M_\Delta kT$$

- The potential energy of the system is simply

$$U \propto \frac{GM_\Delta^2}{R_\Delta}$$

- So we can rewrite the virial theorem ($2K = -U$) as

$$M_\Delta kT \propto \frac{M_\Delta^2}{R_\Delta} \qquad (2.1)$$

MT Relation

$$M_\Delta kT \propto \frac{M_\Delta^2}{R_\Delta} \qquad (2.1)$$

We can express R_Δ in terms of the mean density of the cluster

$$R_\Delta \propto M_\Delta^{1/3} \rho^{-1/3}$$

Substitute into (2.1) and rearrange:

$$M_\Delta \propto (kT)^{3/2} \rho^{-1/2} \qquad (2.2)$$

Now, by definition, the mean density of the cluster within R_Δ is $\Delta\rho_c$ so

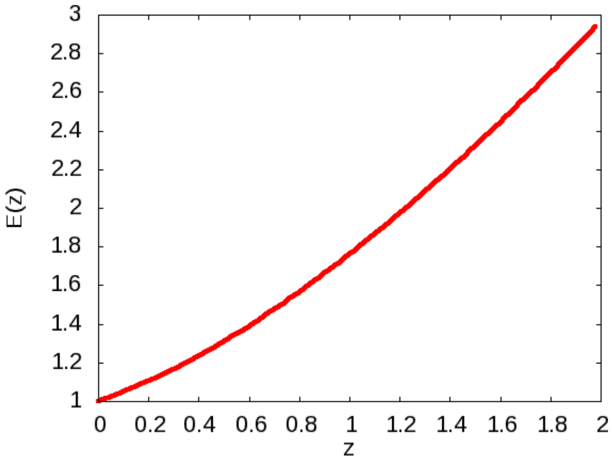
$$\rho = \Delta \rho_c = \Delta \frac{3H^2}{8\pi G}$$

MT Relation

$$\rho = \Delta \rho_c = \Delta \frac{3H^2}{8\pi G}$$

We can describe the redshift-dependence of the Hubble parameter as $H = E(z)H_0$

E(z) is an increasing function of z that depends on cosmological parameters (e.g. Ω_M, Λ)



Then:

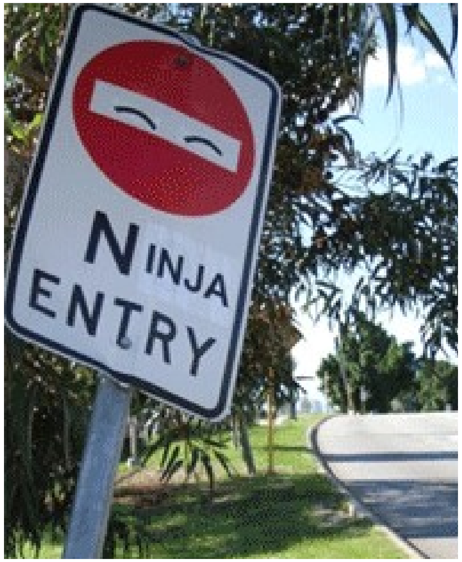
$$\rho \propto \Delta E(z)^2$$

Substitute into (2.2)

$$M_\Delta \propto (kT)^{3/2} \Delta^{-1/2} E(z)^{-1} \qquad (2.3)$$

N.B. Clusters of same mass are hotter at higher z

...and relax



LM Relation

From (1.1), X-ray luminosity from bremsstrahlung

$$L_X \propto \int n_e n_i T^{1/2} dV$$

n_e and n_i are proportional to cluster density ρ for self similar clusters, so write total Lx within R_Δ as:

$$L_X \propto \rho^2 (kT)^{1/2} R_\Delta^3$$

Derive expression for Lx in terms of M, Δ and E(z)

Hint: need to use (2.3)

Example: LM Relation

$$L_X \propto \rho^2 (kT)^{1/2} R_\Delta^3$$

Eliminate R in favour of M and ρ:

$$L_X \propto \rho (kT)^{1/2} M$$

Example: LM Relation

$$L_X \propto \rho^2 (kT)^{1/2} R_\Delta^3$$

Eliminate R in favour of M and ρ:

$$L_X \propto \rho (kT)^{1/2} M$$

Recall $\rho = \Delta \rho_c$ by definition, so:

$$L_X \propto \Delta E(z)^2 (kT)^{1/2} M$$

Example: LM Relation

$$L_X \propto \rho^2 (kT)^{1/2} R_\Delta^3$$

Eliminate R in favour of M and ρ:

$$L_X \propto \rho (kT)^{1/2} M$$

Recall $\rho = \Delta\rho_c$ by definition, so:

$$L_X \propto \Delta E(z)^2 (kT)^{1/2} M$$

Finally, substitute for kT in terms of M, Δ and E(z) from (2.3)

$$L_X \propto \Delta^{7/6} E(z)^{4/3} M^{4/3} \qquad (2.4)$$

Clusters of same M are more luminous at high z

Scaling Relations as Scales

Self-similar model predicts scaling relations between easily measured properties and cluster mass

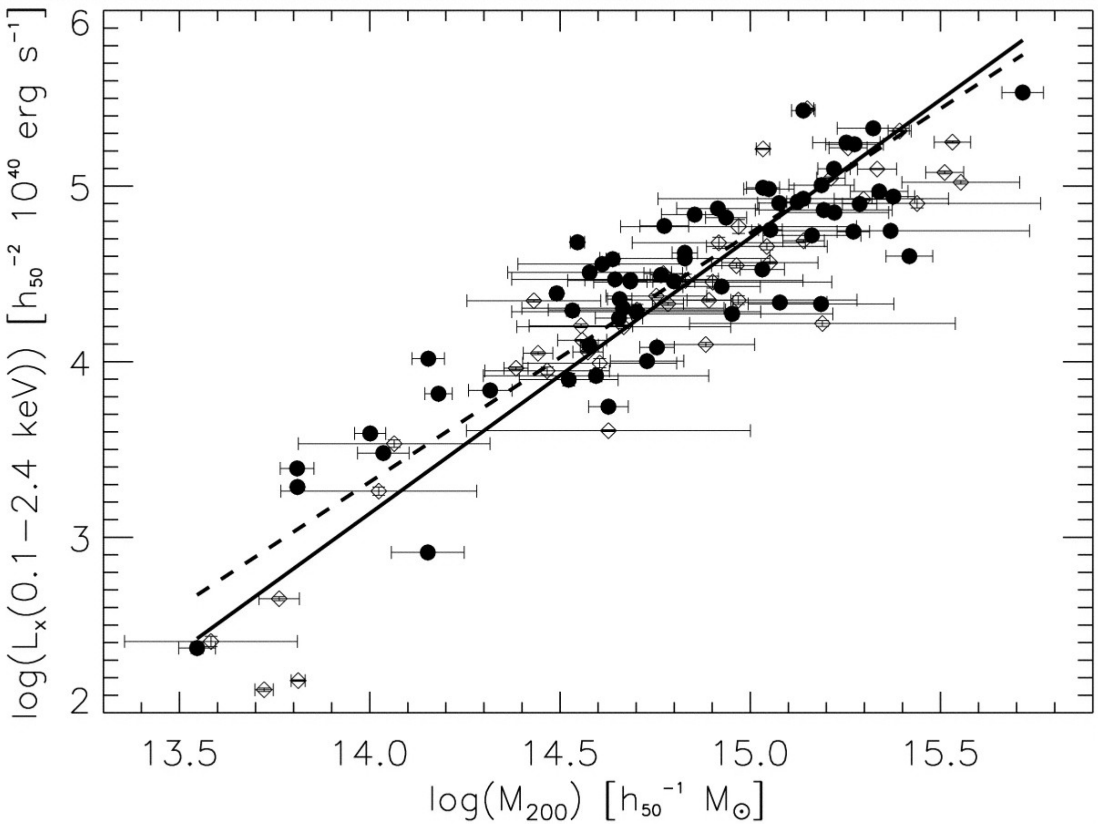
★ Determining mass of cluster difficult

★ Scaling relations allow masses to be estimated from easy to measure properties

★ Measure masses for **large samples** of **distant** clusters with **lower quality** data

- Cosmological studies

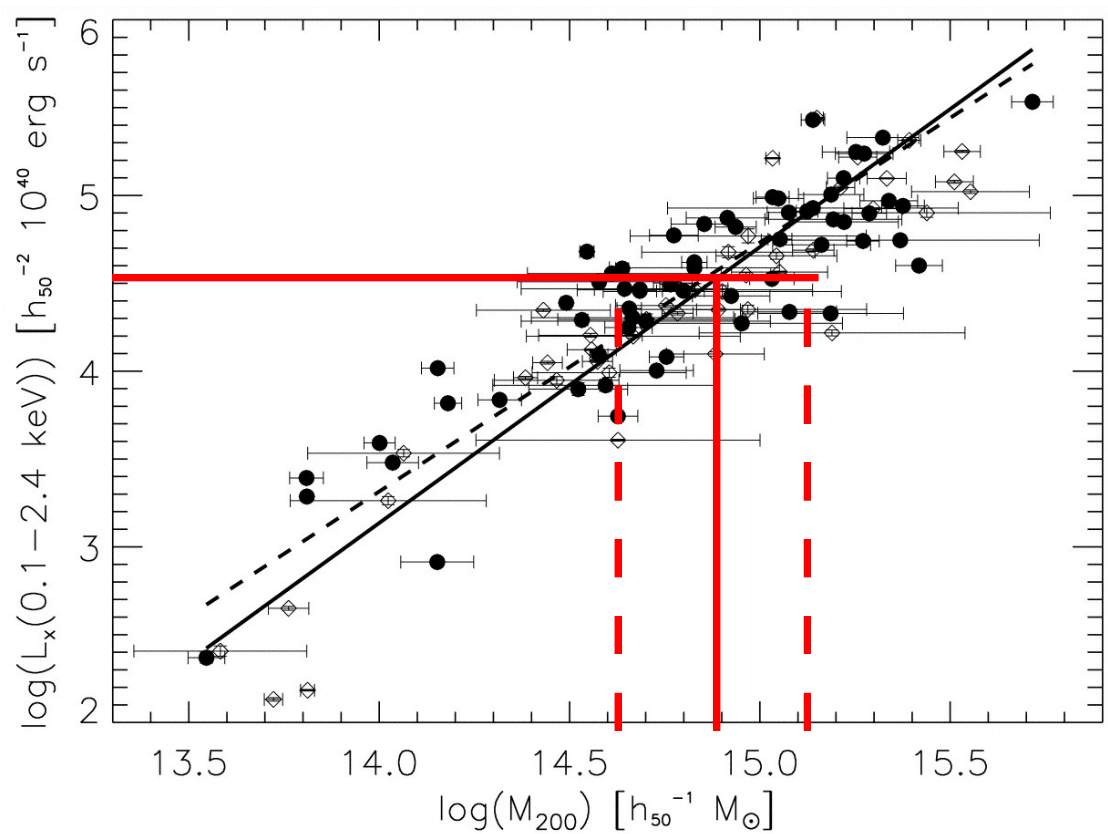
(LM relation from Reiprich & Bohringer 2002, ApJ, 567)



Scaling Relations as Scales

- ★ *Precision* of the "scales" depends on the intrinsic scatter
  - scatter beyond that expected from errors
  - due to real cluster-to-cluster variation
  - cannot beat with longer observations

- ★ *Accuracy* of "scales" depends on calibration of relations
  - measure with hydrostatic X-ray masses or lensing

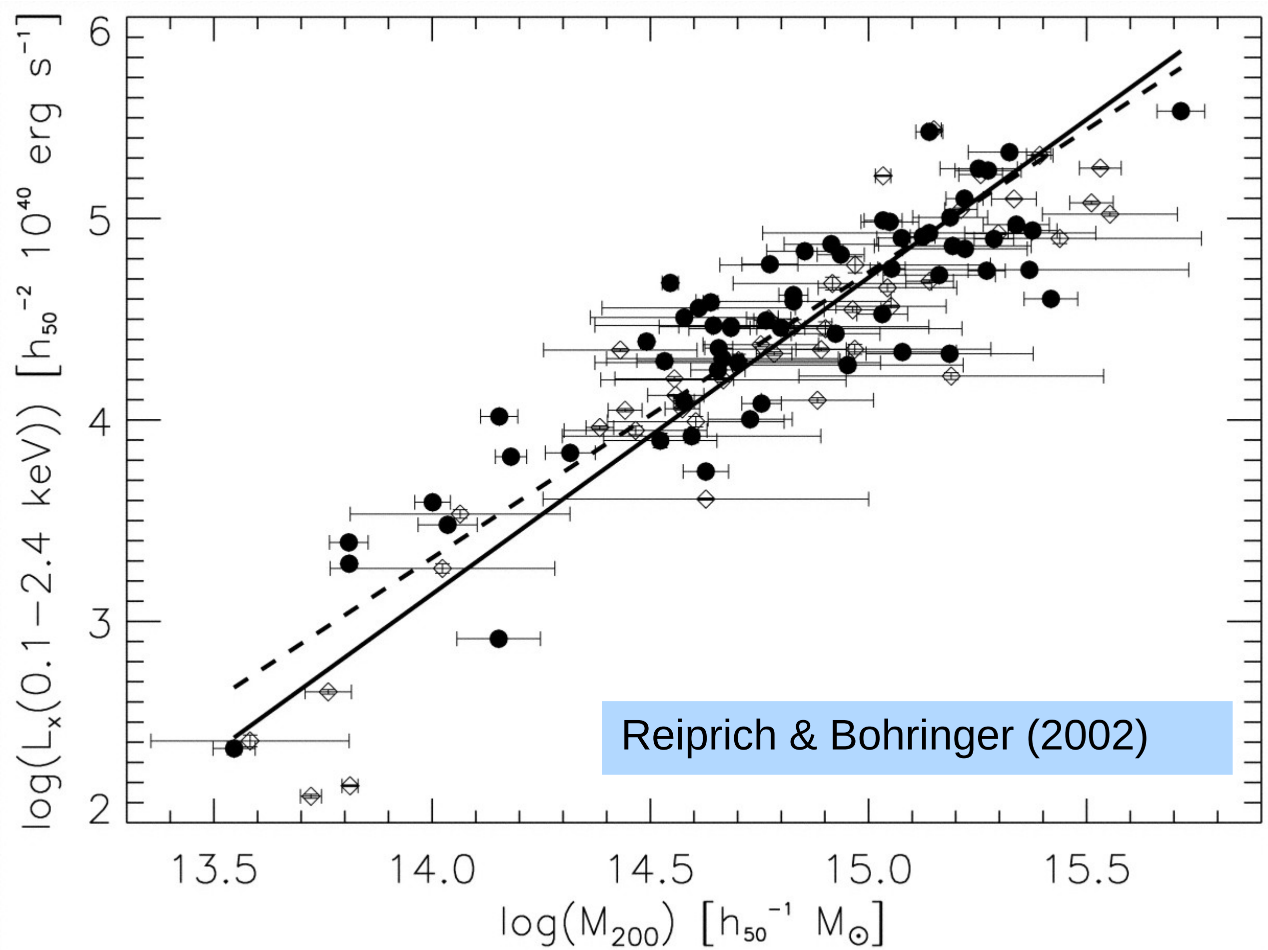


- ★ Ideally want scaling relation with lowest intrinsic scatter

Lx as a mass proxy

Lx is **easiest** property to measure

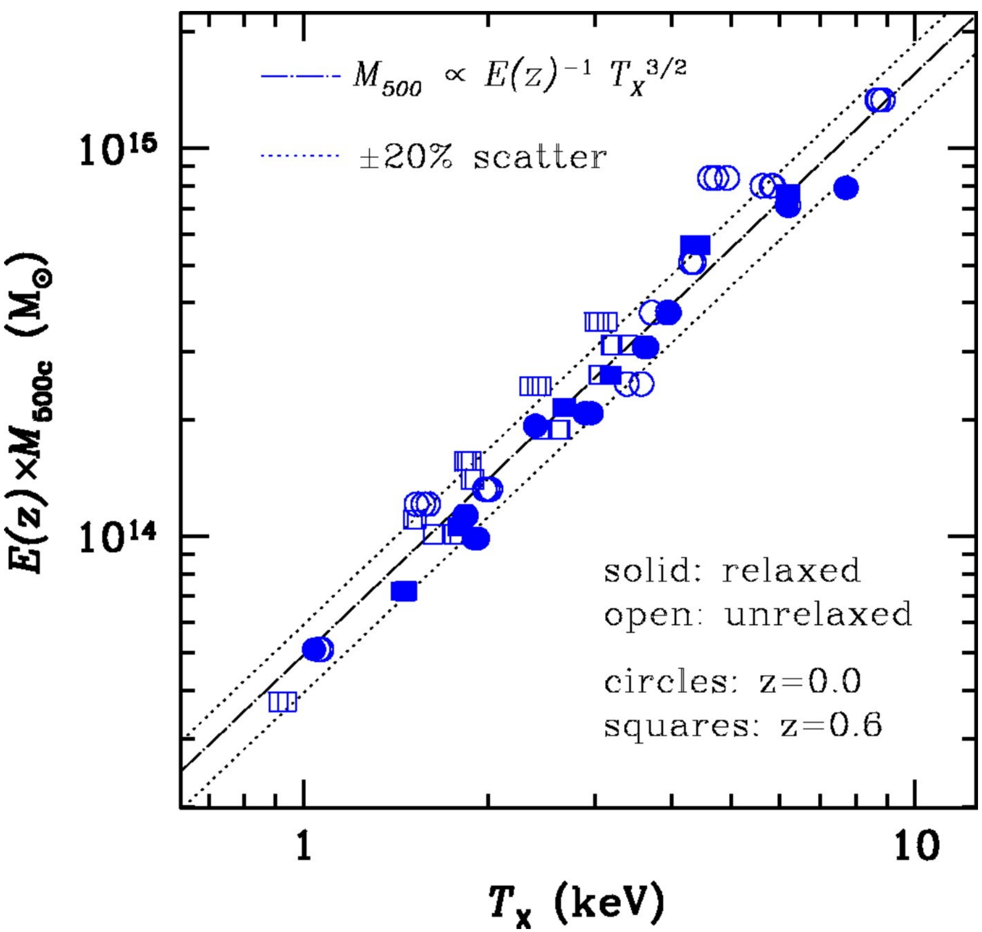
- ★ Early work showed large scatter with mass (~60%)
- ★ Recent measures suggest closer to 40% (Maughan 2007)



kT as a mass proxy

kT has a fairly tight scaling relation with M for **relaxed** clusters

★ **Merging** clusters add scatter and systematic uncertainty



★ Simulations show ~20% scatter in MT relation

★ N.B. Simulations extremely helpful as we know true mass of clusters

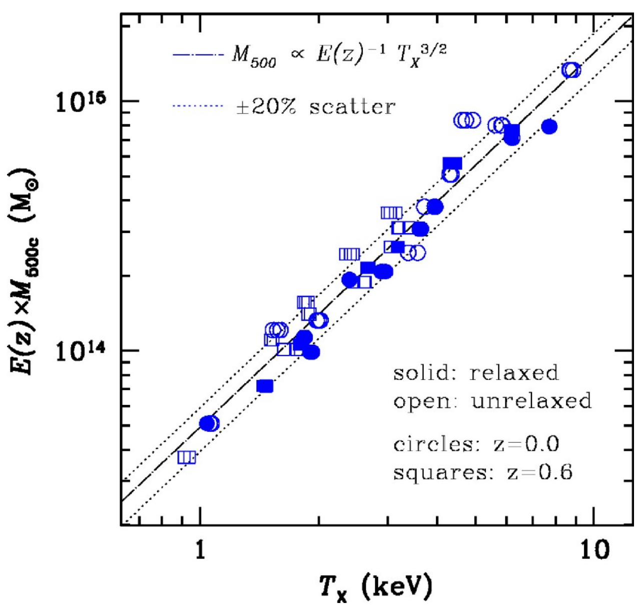
★ Systematic offset between relaxed ■ and merging □ clusters

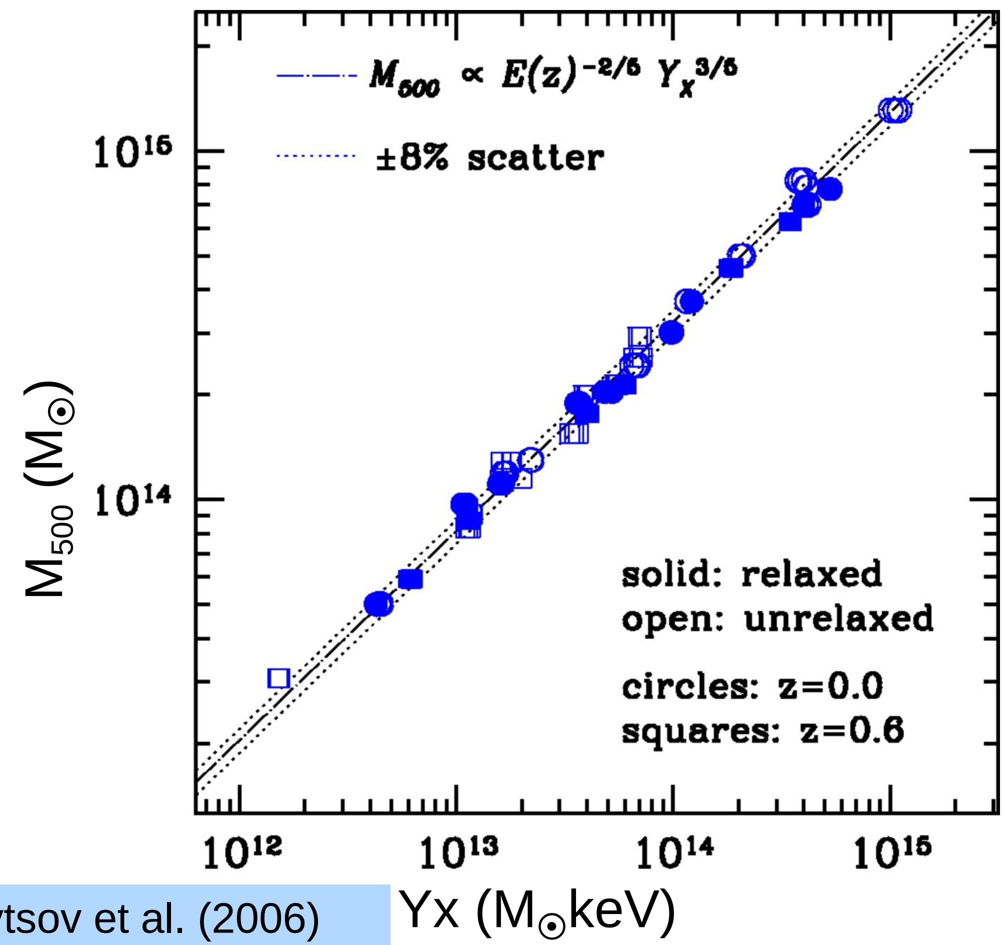
Kravtsov et al. (2006, ApJ, 650)

Yx – a super scaler!

Recent work has shown **Yx is superior mass indicator**

- **Product of kT and M_{gas}** (both easily measured) within R500 with central $0.15R_{500}$ excluded
- Just **8%** scatter with mass
- **Insensitive to mergers** (no offset between relaxed and merging clusters)





Kravtsov et al. (2006)

Summary I

Self-similar model assumes:

- ★ Clusters form in single collapse at z_{obs}
- ★ Gravity only source of energy

Self-similar model predicts:

- ★ Clusters of different masses are scaled versions
- ★ Clusters at different z identical if scaled for $\rho_c(z)$

Define cluster properties within overdensity radii

- ★ Mean density enclosed is Δ times $\rho_c(z)$
- ★ Fair comparison of clusters of different M and z

Summary II

Derive self-similar scaling relations

- ★ Simple power laws relating cluster properties
- ★ MT, LM, LT etc

Scaling relations have potential to allow estimation of cluster masses from easily measured properties

- ★ Precision depends on intrinsic scatter
- ★ Accuracy depends on calibration (which masses to use)
- ★ Lx – kT – Yx increasingly precise mass proxies

Hydrostatic masses most reliable

- ★ need high quality data for T(r) and ρ(r)
- ★ need relaxed clusters