V. The Cluster Mass Function
Baryon Fraction Summary

Assuming clusters large enough to be representative, mass composition should match Universe

$$f_b = \frac{M_b}{M_{tot}} = \frac{\Omega_b}{\Omega_M}$$

★ observe $f_b$ and constrain $\Omega_M$

Assuming $f_b$ redshift independent, any observed variation with $z$ due to assumed cosmology

$$f_{gas} = \propto d_L d_A^{1/2}$$

★ constrain $E(z)$ and from observed $f_b(z)$
★ combined with CMB and SNIa and including possible sytematics:

$$\Omega_M = 0.253 \pm 0.021 \quad w = -0.98 \pm 0.07$$
Recall that initially overdense regions overcome expansion to collapse to form structures

Structure in Universe depends on
★ expansion history: \( E(z) \)
★ initial density distribution: \( \sigma_8 \)

Number density of clusters sensitive to growth of structure
★ also sensitive to volume sampled
★ additional \( E(z) \) constraints

\[
dV_{\chi}(z) = \frac{c}{a_0 H_0} \frac{(1 + z)^2 d_A^2}{E(z)} d\Omega d\ell
\]
Mass Function

Mass function describes number of clusters of mass $M$ per unit comoving volume

- can be derived analytically, but most commonly measured from large volume simulations
- simulate volume of Universe and detect and count structures of different mass at different $z$
- repeat for different cosmologies

Jenkins et al. (2001; MNRAS, 321)
Mass function describes number of clusters of mass $M$ per unit comoving volume

- decreasing function of $M$
- steepens at high $M$
  - very high mass clusters extremely rare

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  - shape of MF at $z=0$

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- changing cosmological parameters affects:
  - shape of MF at $z=0$
  - evolution of MF with redshift

Obtain cosmological constraints by counting $n(M)$ for clusters at different $z$

Measuring the Mass Function

To measure the MF observationally, need three stages

★ detect and count clusters
  ▶ cluster surveys

★ determine volume surveyed
  ▶ survey selection function

★ estimate cluster masses
  ▶ scaling relations
Cluster Surveys

As we saw, clusters first detected in optical
★ prone to projection effects
★ red sequence surveys promising
  ▶ select clusters based on galaxies of same colour

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**prone to projection effects**
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Weak lensing surveys being developed

SZ surveys promising due to $z$ independence
**Vanderlinde et al (2010; ApJ 722) for early results**
Serendipitous X-ray surveys currently most successful
☆ look at archive of X-ray images of compact targets
☆ detect clusters as extended sources in X-ray images

☆ follow up optical images to confirm galaxies
☆ optical spectra to measure redshift – confirmed cluster

Detection of a cluster depends on X-ray surface brightness
★ flux / solid angle
★ high SB – compact source – high contrast against background – easy to detect
★ low SB – diffuse source – low contrast – hard to detect
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SB depends on:
★ flux – depends on L and z (or M and z) – dominates
★ angular size – depends on physical size and z – we'll neglect this

Typically define flux-limited sample
★ i.e. detected all clusters brighter than $F_{\text{lim}}$ in survey area
Cluster Masses

X-ray survey lets us count clusters and measure $F$ and $z$

★ mass function needs number **density** of clusters in each **mass** bin ($M +/\Delta M$)
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* mass function needs number **density** of clusters in each **mass** bin \((M \pm \Delta M)\)

To get masses:
* use F, z to give L
* use LM relation for M
Cluster Masses

X-ray survey lets us count clusters and measure $F$ and $z$

★ mass function needs number **density** of clusters in each **mass** bin ($M \pm \Delta M$)

To get masses:

★ use $F$, $z$ to give $L$
★ use LM relation for $M$

or

★ follow up X-ray observations to measure $kT$, $Y_x$
★ use MT or MY relations
Survey Volume

To compute number density, need survey volume
★ suppose we survey solid angle $\Omega$ on sky
★ detect $n$ clusters in some mass bin $(M +/- \Delta M)$

Q: what volume do we use to get density?

$$dV_x(z) = \frac{c}{a_0 H_0} \frac{(1 + z)^2 d^2 A}{E(z)} d\Omega dz$$

Integrate $dV$ over $\Omega$ from $z=0$ to $z_{\text{max}}$
★ how decide what $z_{\text{max}}$?
$z_{\text{max}}$ is max redshift to which we could have detected a cluster
★ depends on L and hence M of cluster
★ at some z, model a cluster of mass M
★ calculate L for that M (LM relation)
★ calculate flux for that L, z (cosmology dependent)
★ $z_{\text{max}}$ is when flux drops below flux limit
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$z_{\text{max}}$ depends on mass of cluster considered
★ $z_{\text{max}}(M)$
Survey Volume

Ω is survey area – also depends on cluster mass
★ survey area made up of many X-ray fields
★ sensitivity of fields not uniform
   ▶ different exposure times
   ▶ highest sensitivity in centre
★ bright sources could be detected near edge of field
★ faint sources only detected near centre or longer exposures
   ▶ smaller survey area

Chandra exposure map
Survey Volume

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Survey area depends on source flux
★ i.e. depends on mass and redshift
★ $\Omega(M, z)$
Survey Volume

Integrate volume element to get survey volume

\[ V(M) \sim \int_0^{z_{\text{max}}(M)} \Omega(M, z) \, dz \]

- area and \( z_{\text{max}} \) both depend on \( M \)
  - volume surveyed depends on \( M \)
  - larger survey volume for more massive clusters
  - brighter and so easier to detect

- calculated \( V \) depends on LM relation and cosmology
  - often written as \( V(L) \)
  - referred to as selection function
“The other two are still lost on the infinite plane of uniform density”
- xkcd.com
Henry & Arnaud (1991) used temperature function of 25 clusters at $z<0.1$

\[
\sigma_8 = 0.59 \pm 0.02
\]
Reiprich & Bohringer (2002) used 63 clusters at $z<0.1$ to measure mass function:

$$\Omega_M = 0.12 \pm 0.05, \sigma_8 = 0.96 \pm 0.14$$

★ N.B. $\Omega_M$ and $\sigma_8$ are anti-corellated
More recently, Vikhlinin et al (2009; ApJ 692) used 37 clusters at $<z>=0.55$ and 49 clusters at $<z>=0.05$

- Taken from 400SD X-ray cluster survey
- Clusters reobserved with Chandra for high quality data
- Used Yx scaling relation to estimate cluster masses
More recently, Vikhlinin et al (2009; ApJ 692) used 37 clusters at $<z>=0.55$ and 49 clusters at $<z>=0.05$

Note how predicted function and measured values are both sensitive to cosmology for high-$z$ clusters
Experimental Results

Vikhlinin's constraints on $\Omega_M$ and $\sigma_8$

- different techniques give range in $\sigma_8 - \Omega_M$ plane
- this work: $\sigma_8 = 0.813 \pm 0.012$
- simulations depend on $\sigma_8$ - like higher values as get more clusters!

Reiprich+ 2002

WMAP 3yr & 5yr
Experimental Results

Vikhlinin's constraints on $\Omega_\Lambda$ and $w$

- assumed flat Universe here
- note improvement of adding clusters
- from cluster mass function alone: $w = -1.14 \pm 0.21$
  - recall cluster $f_{\text{gas}}(z)$: $w = -1.14 \pm 0.31$

combined constraints:

$$w = -0.991 \pm 0.045$$

$$\Omega_\Lambda = 0.740 \pm 0.012$$
Caution: Mass Accuracy

Dominant source of error is mass scaling relations
★ LM relation for volume calculations
★ YM relation (or MT etc) for mass estimates

★ good agreement, but more precision required
★ tests of evolution of mass scaling relations needed
Caution: Mass Accuracy

Recall ~10% underestimate of X-ray masses c.f. simulations

* black contour shows effect on $\sigma_8, \Omega_M$

How well are selection functions known?

★ Santos et al (2010; A&A) compared surface brightness concentration $c_{SB}$ for different high-z X-ray samples.

Found significant difference in distributions

★ 400SD (Vikhlinin) survey missing concentrated clusters at high-z

★ clusters misclassified as point sources?

★ errors in selection function?
High mass clusters are
★ brighter & rarer than low mass
★ $z_{\text{max}}$ larger for high mass clusters
  ▶ survey volume much larger
Caution: Selection Biases

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Few low mass clusters detected at high-z as too faint
Few high mass clusters detected at low-z as too rare (small volume)
★ Mean $z$ of massive clusters higher than low-mass clusters
★ **Malmquist bias** – accounted for by selection function
Caution: Selection Biases

Consider flux limited sample at some $z$

- flux limit corresponds to some mass from LM relation
- Scatter in $L(M)$ means some clusters with masses too low will be in sample and vice-versa
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- Amount of bias depends on slope at limit & scatter
Caution: Selection Biases

Known as **Eddington Bias**
★ overestimate cluster masses and number densities

Can correct if scatter in LM is known
★ does scatter vary with z?
★ hard to measure, but may decrease with z
Cluster mass function sensitive to cosmology through
★ growth of structure – N(M)
★ geometry – d(z), V(z)

Large, well-calibrated X-ray samples measure shape and evolution of MF
★ selection function gives V(M,L)
★ best constraints from reobserving clusters to get T, Yx

Mass uncertainties dominant source of error
★ affect M and V calculations

Selection function essential to control biases
Clusters powerful cosmological probes, with different sensitivities, assumptions to other methods

Current best bet:
★ flat Universe, 70% dark energy
★ DE is in form of cosmological constant (w=-1)

combined constraints:
\[ w = -0.991 \pm 0.045 \]
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