V. The Cluster Mass Function

Baryon Fraction Summary

Assuming clusters large enough to be representative, mass composition should match Universe

$$f_b = \frac{M_b}{M_{tot}} = \frac{\Omega_b}{\Omega_M}$$

★ observe f_b and constrain Ω_M

Assuming f_b redshift independent, any observed variation with z due to assumed cosmology

$$f_{gas} = \propto d_L d_A^{1/2}$$

★ constrain E(z) and from observed $f_b(z)$

★ combined with CMB and SNIa and including possible sytematics:

$$\Omega_M = 0.253 \pm 0.021 \qquad w = -0.98 \pm 0.07$$

Structure Growth Recap

Recall that initially overdense regions overcome expansion to collapse to form structures

Structure in Universe depends on

★ expansion history: E(z)

★ initial density distribution: σ_8

Number density of clusters sensitive to growth of structure

★ also sensitive to volume sampled

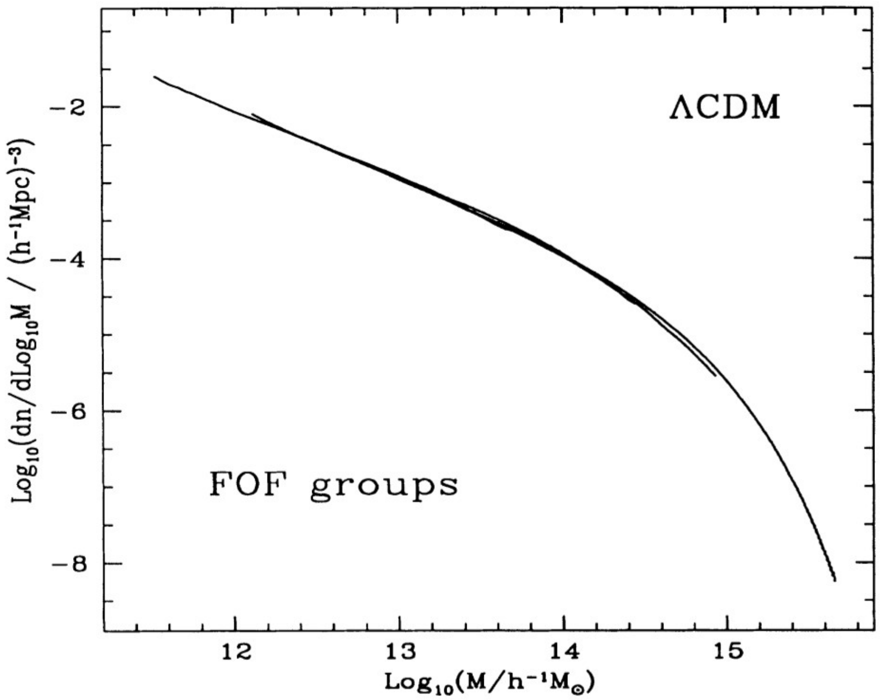
★ additional E(z) constraints

$$dV_\chi(z) = \frac{c}{a_0 H_0} \frac{(1+z)^2 d_A^2}{E(z)} d\Omega dz$$

Mass Function

Mass function describes number of clusters of mass M per unit comoving volume

- ⋆ can be derived analytically, but most commonly measured from large volume simulations
- ⋆ simulate volume of Universe and detect and count structures of different mass at different z
- ⋆ repeat for different cosmologies

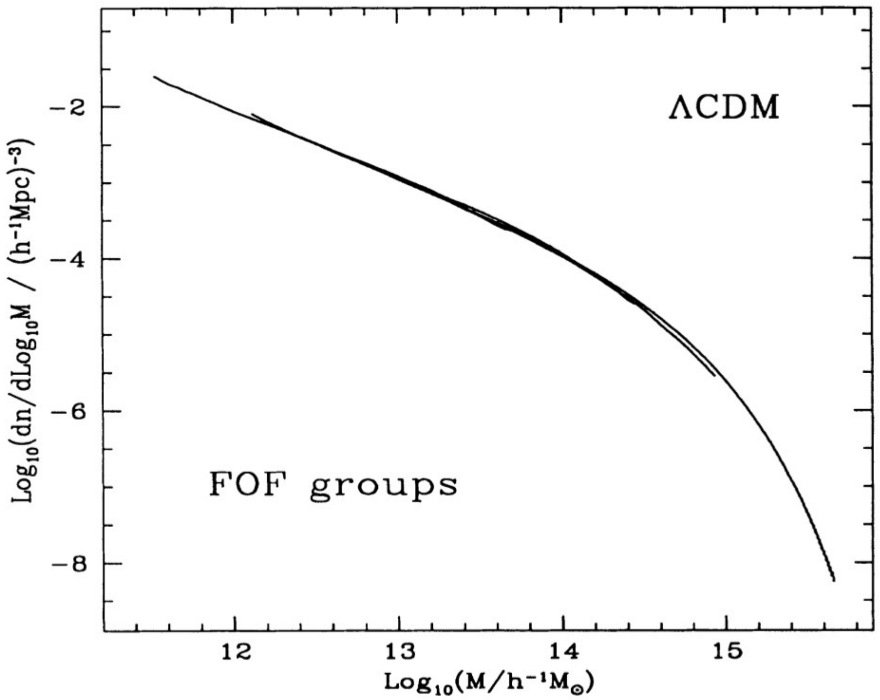


Jenkins et al. (2001; MNRAS, 321)

Mass Function

Mass function describes number of clusters of mass M per unit comoving volume

- ★ decreasing function of M
- ★ steepens at high M
  - ▸ very high mass clusters extremely rare

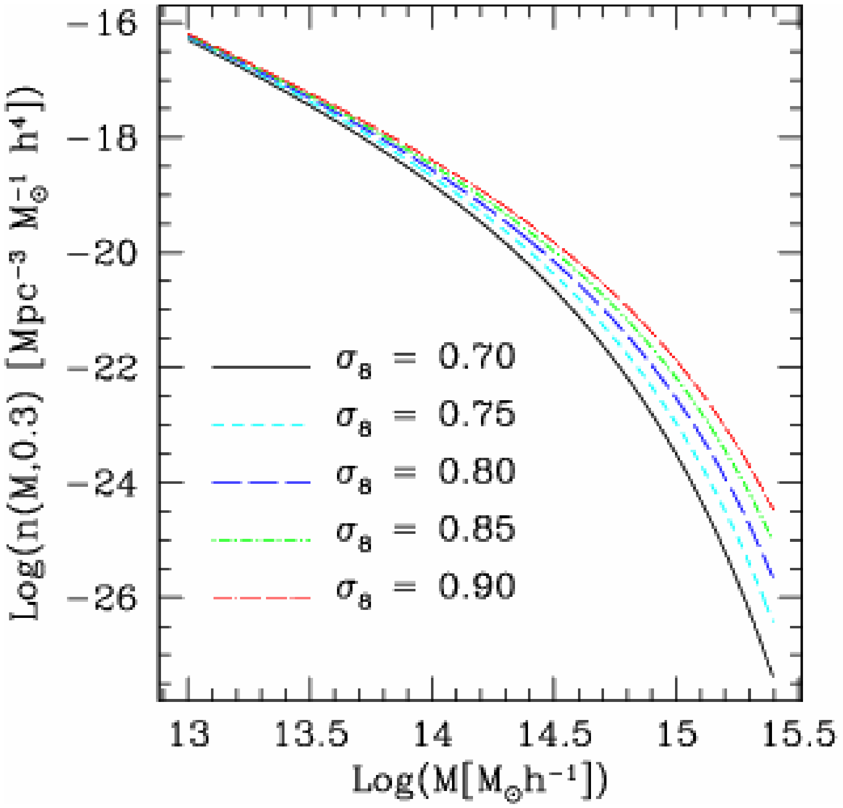


Jenkins et al. (2001; MNRAS, 321)

Mass Function

Mass function describes number of clusters of mass M per unit comoving volume

- ★ changing cosmological parameters affects:
  - ▸ shape of MF at z=0



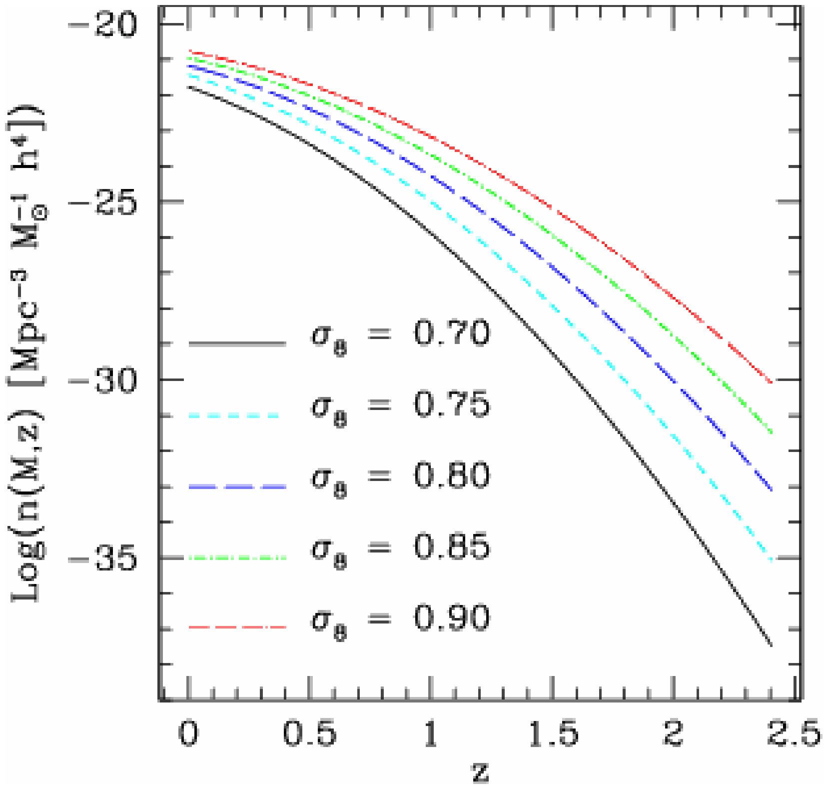
Fedeli et al, (2008, A&A, 486)

Mass Function

Mass function describes number of clusters of mass M per unit comoving volume

- ★ changing cosmological parameters affects:
  - ▸ shape of MF at z=0
  - ▸ evolution of MF with redshift

Obtain cosmological constraints by counting n(M) for clusters at different z



Fedeli et al, (2008, A&A, 486)

Measuring the Mass Function

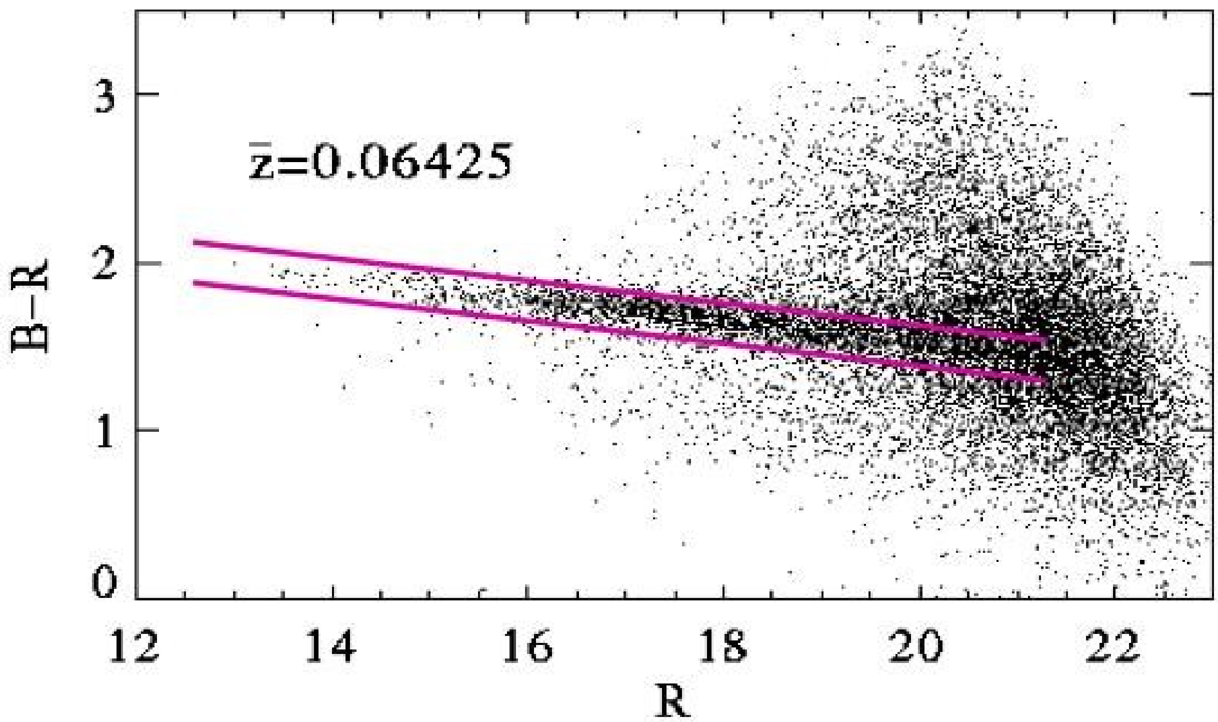
To measure the MF observationally, need three stages

- ★ detect and count clusters
  - ▸ cluster surveys
- ★ determine volume surveyed
  - ▸ survey selection function
- ★ estimate cluster masses
  - ▸ scaling relations

Cluster Surveys

As we saw, clusters first detected in optical

- ★ prone to projection effects
- ★ red sequence surveys promising
  - ▸ select clusters based on galaxies of same colour



Cluster Surveys

As we saw, clusters first detected in optical

- ★ prone to projection effects
- ★ red sequence surveys promising
  - ▸ select clusters based on galaxies of same colour

Weak lensing surveys being developed

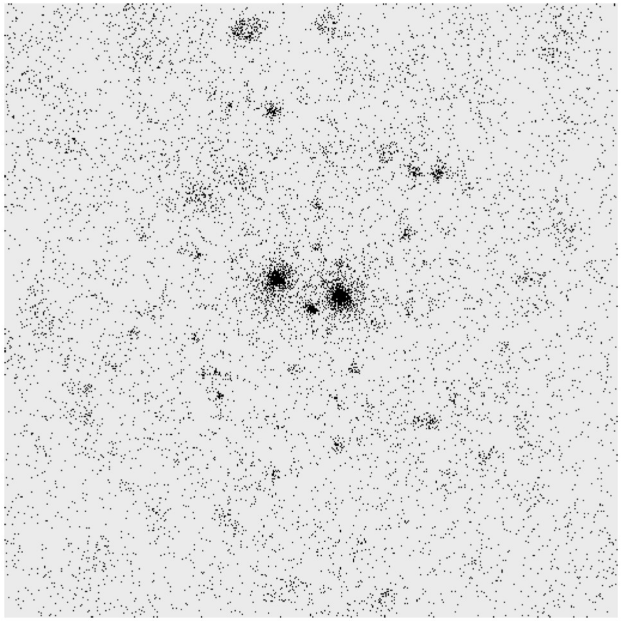
SZ surveys promising due to z independence

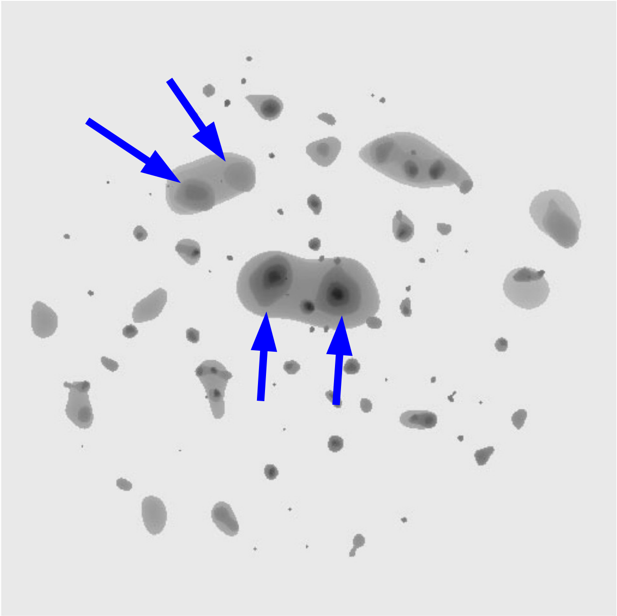
- ★ Vanderlinde et al (2010; ApJ 722) for early results

X-ray Surveys

Serendipitous X-ray surveys currently most successful

- ⋆ look at archive of X-ray images of compact targets
- ⋆ detect clusters as extended sources in X-ray images



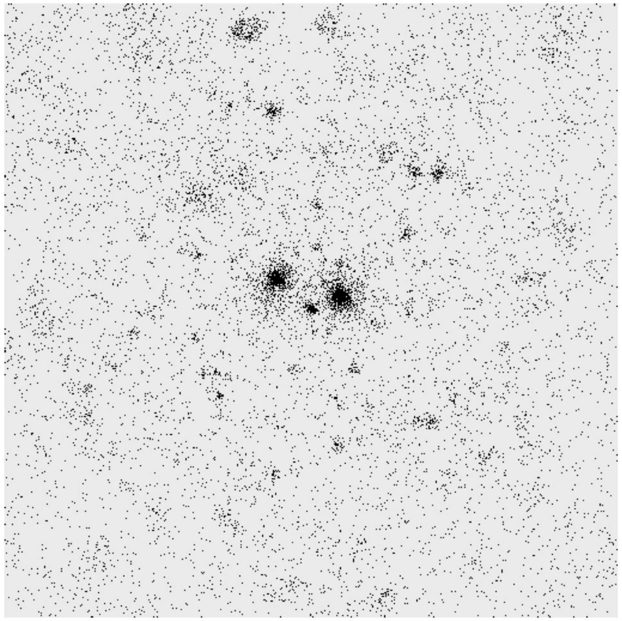


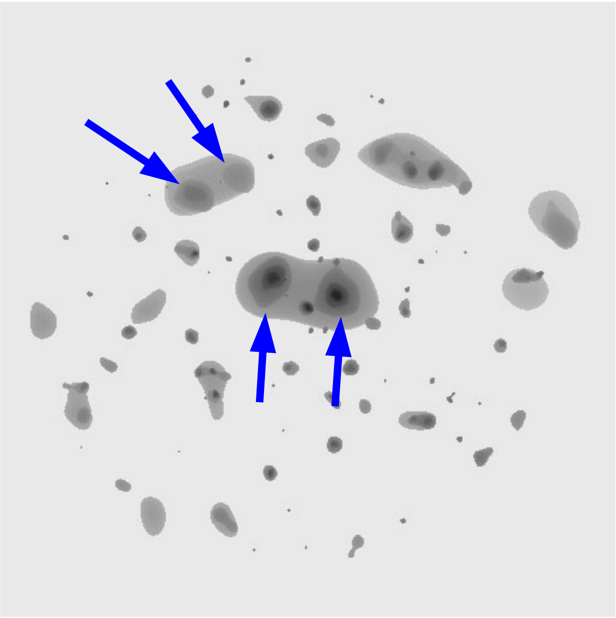
- ⋆ follow up optical images to confirm galaxies
- ⋆ optical spectra to measure redshift – confirmed cluster

X-ray Surveys

Detection of a cluster depends on X-ray surface brightness

- ★ flux / solid angle
- ★ high SB – compact source – high contrast against background – easy to detect
- ★ low SB – diffuse source – low contrast – hard to detect





X-ray Surveys

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SB depends on:

- ★ flux – depends on L and z (or M and z) – dominates
- ★ angular size – depends on physical size and z – we'll neglect this

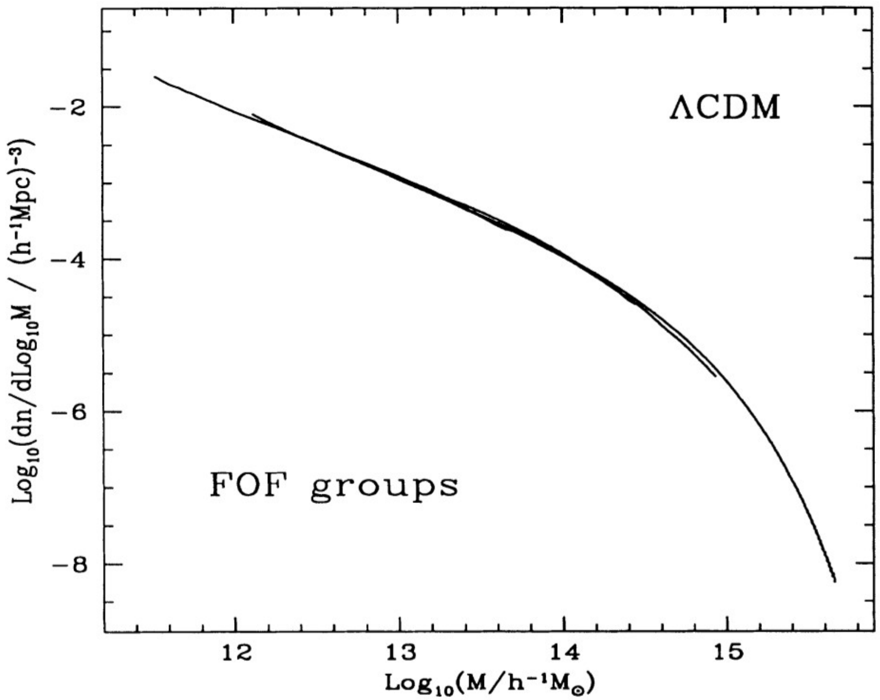
Typically define flux-limited sample

- ★ i.e. detected all clusters brighter than F_{lim} in survey area

Cluster Masses

X-ray survey lets us count clusters and measure F and z

- ★ mass function needs number **density** of clusters in each **mass** bin (M +/- ΔM)



Cluster Masses

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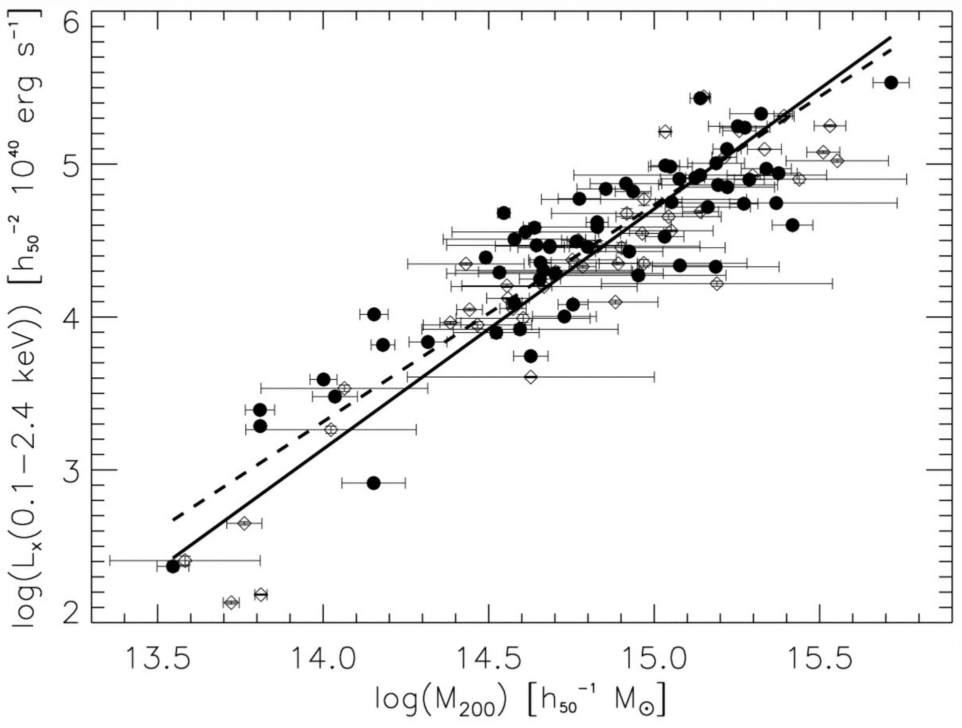
- ★ mass function needs number **density** of clusters in each **mass** bin (M +/- ΔM)

To get masses:

- ★ use F, z to give L
- ★ use LM relation for M



Cluster Masses

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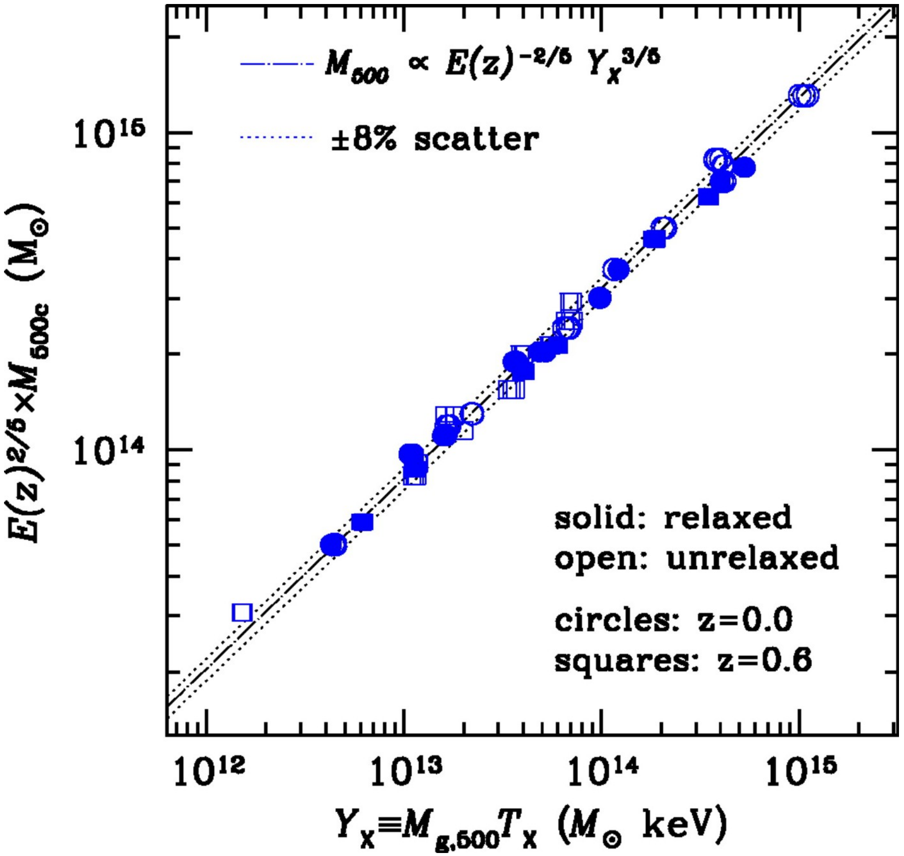
- ★ mass function needs number **density** of clusters in each **mass** bin (M +/- ΔM)

To get masses:

- ★ use F, z to give L
- ★ use LM relation for M

or

- ★ follow up X-ray observations to measure kT, Yx
- ★ use MT or MY relations



Survey Volume

To compute number density, need survey volume

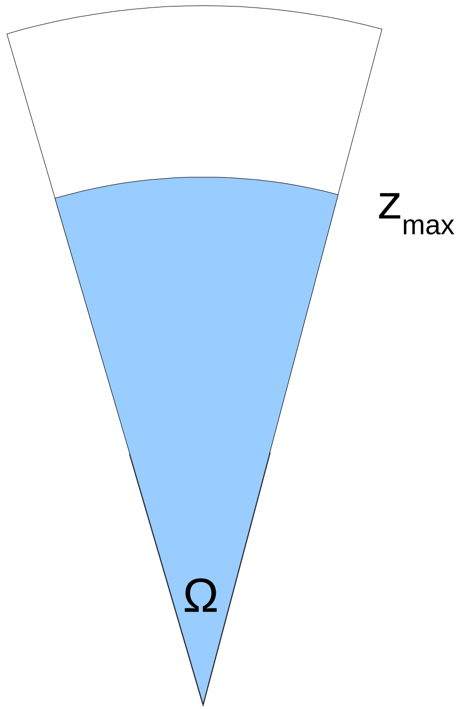
- ⋆ suppose we survey solid angle Ω on sky
- ⋆ detect n clusters in some mass bin (M +/- ΔM)

Q: what volume do we use to get density?

$$dV_\chi(z) = \frac{c}{a_0 H_0} \frac{(1+z)^2 d_A^2}{E(z)} d\Omega dz$$

Integrate dV over Ω from z=0 to z_{max}

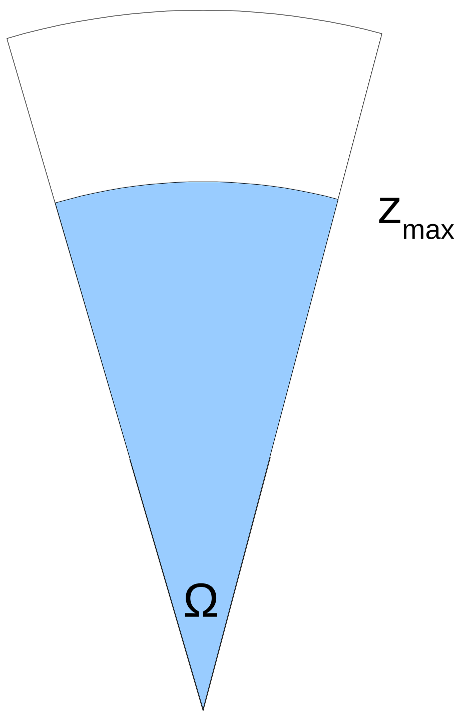
- ⋆ how decide what z_{max}?



Survey Volume

z_{max} is max redshift to which we could have detected a cluster

- ★ depends on L and hence M of cluster
- ★ at some z, model a cluster of mass M
- ★ calculate L for that M (LM relation)
- ★ calculate flux for that L, z (cosmology dependent)
- ★ z_{max} is when flux drops below flux limit



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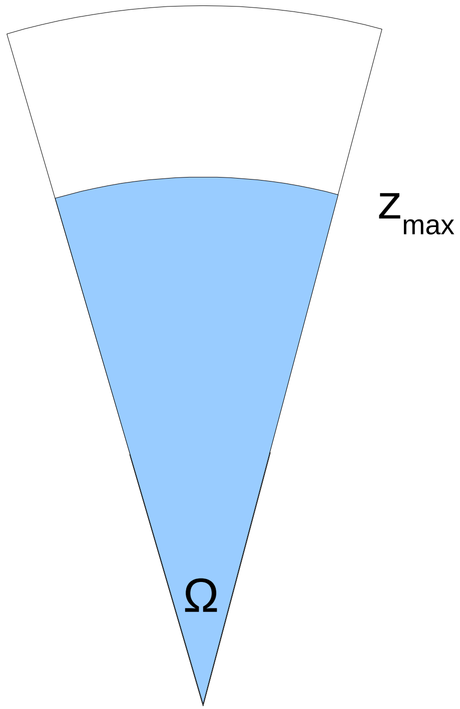


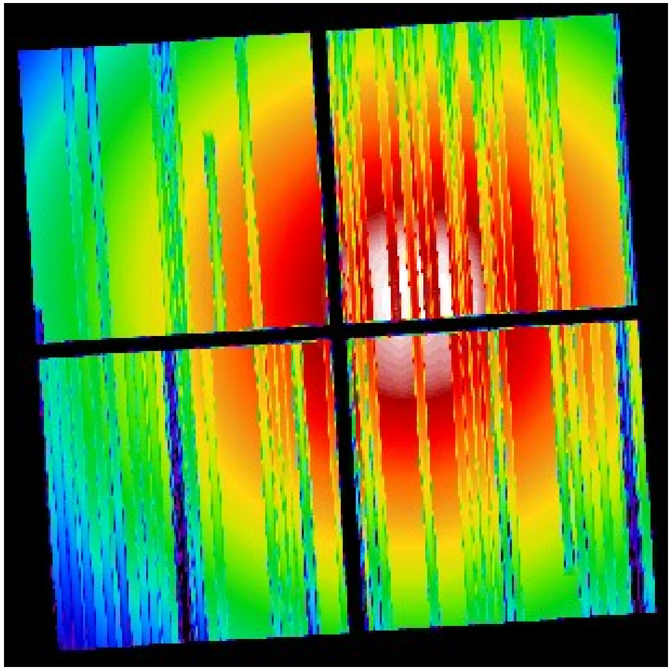
z_{max} depends on mass of cluster considered

- $z_{max}(M)$

Survey Volume

Ω is survey area – also depends on cluster mass

- ⋆ survey area made up of many X-ray fields
- ⋆ sensitivity of fields not uniform
  - ▸ different exposure times
  - ▸ highest sensitivity in centre
- ⋆ bright sources could be detected near edge of field
- ⋆ faint sources only detected near centre or longer exposures
  - ▸ smaller survey area



Chandra exposure map

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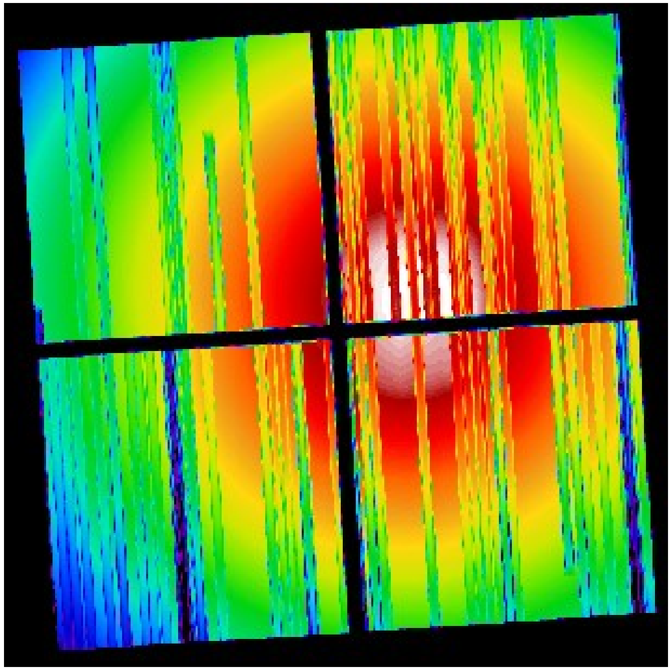


Chandra exposure map

Survey area depends on source flux

- i.e. depends on mass and redshift
- Ω(M,z)

Survey Volume

Integrate volume element to get survey volume

$$V(M) \sim \int_0^{z_{max}(M)} \Omega(M, z) dz$$

- ★ area and z_{max} both depend on M
  - ▸ volume surveyed depends on M
  - ▸ larger survey volume for more massive clusters
  - ▸ brighter and so easier to detect

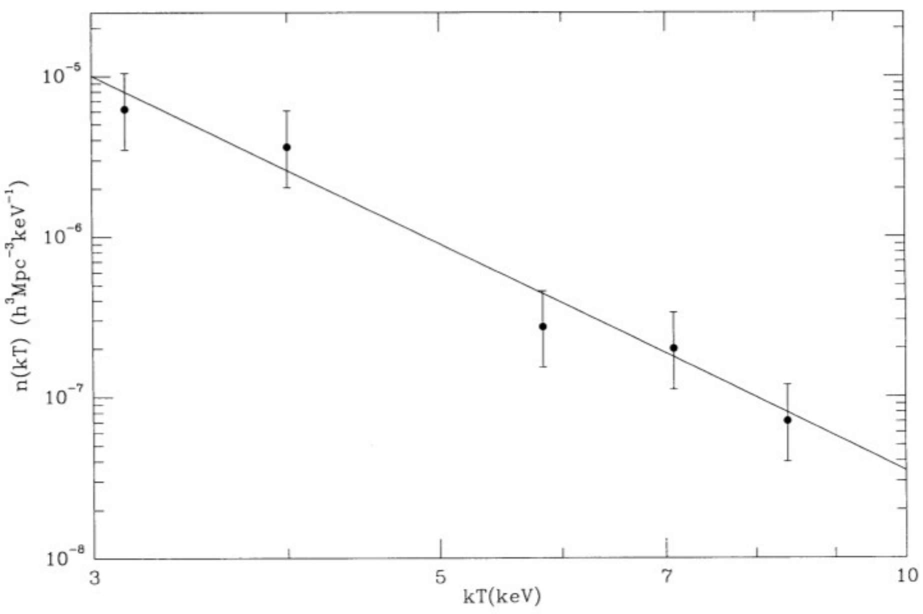
- ★ calculated V depends on LM relation and cosmology
  - ▸ often written as V(L)
  - ▸ referred to as selection function

xkcd break

“The other two are still lost on the infinite plane of uniform density”
- xkcd.com

Experimental Results

Henry & Arnaud (1991) used temperature function of 25 clusters at z<0.1

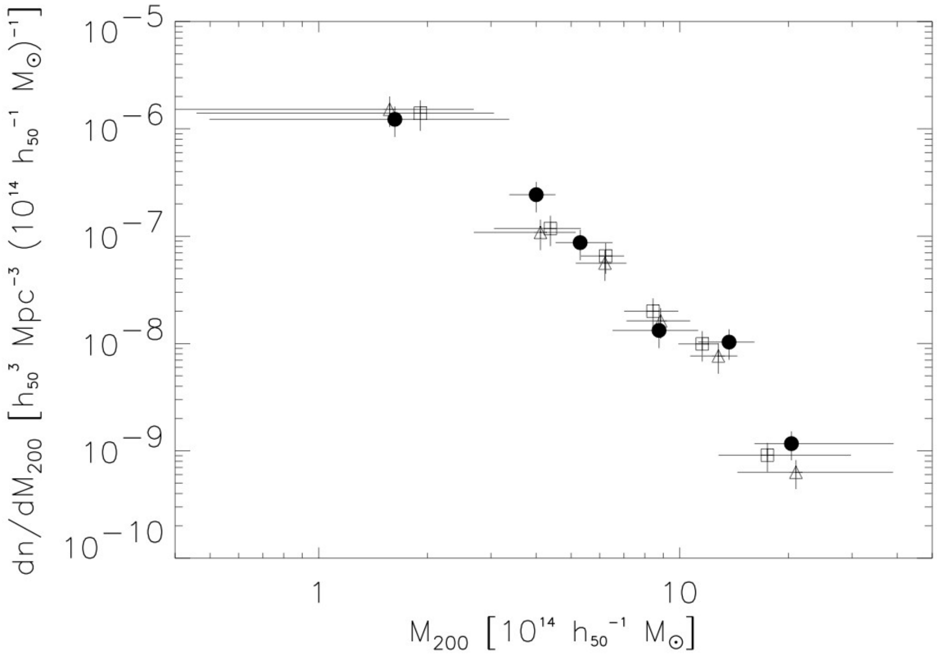


- ★ Temp function related to mass function by MT relation

$$\sigma_8 = 0.59 \pm 0.02$$

Experimental Results

Reiprich & Bohringer (2002) used 63 clusters at z<0.1 to measure mass function

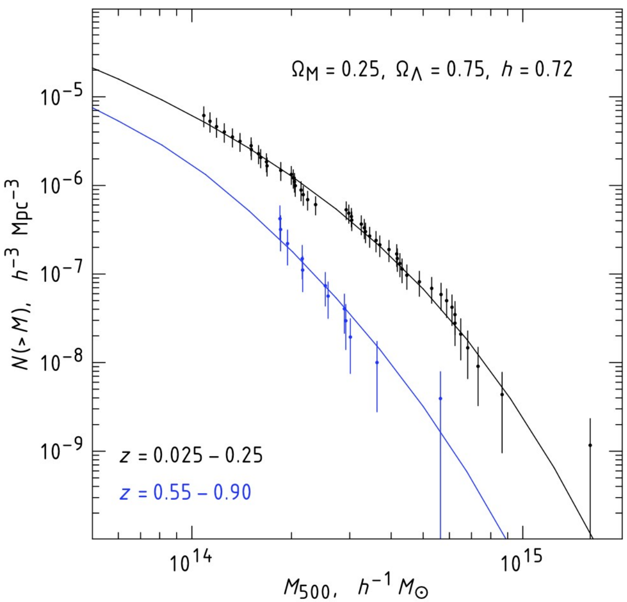


$$\Omega_M = 0.12 \pm 0.05, \sigma_8 = 0.96 \pm 0.14$$

★ N.B. Ω_M and σ_8 are anti-corellated

Experimental Results

More recently, Vikhlinin et al (2009; ApJ 692) used 37 clusters at <z>=0.55 and 49 clusters at <z>=0.05



- ⋆ Taken from 400SD X-ray cluster survey
- ⋆ Clusters reobserved with Chandra for high quality data
- ⋆ Used Yx scaling relation to estimate cluster masses

Experimental Results

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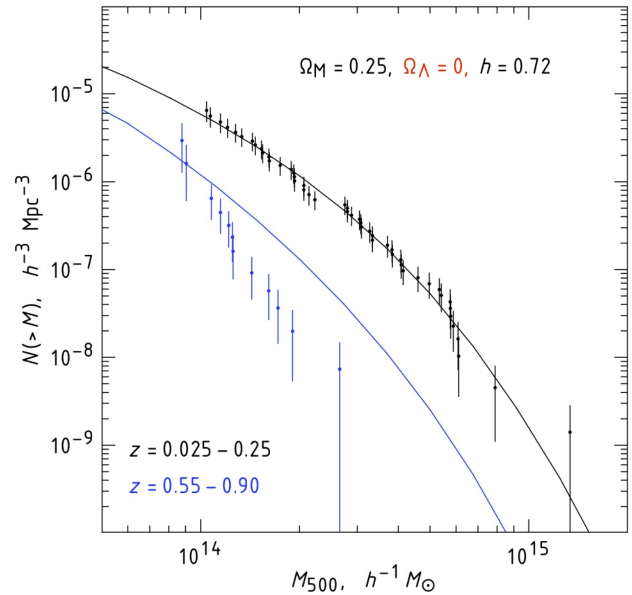
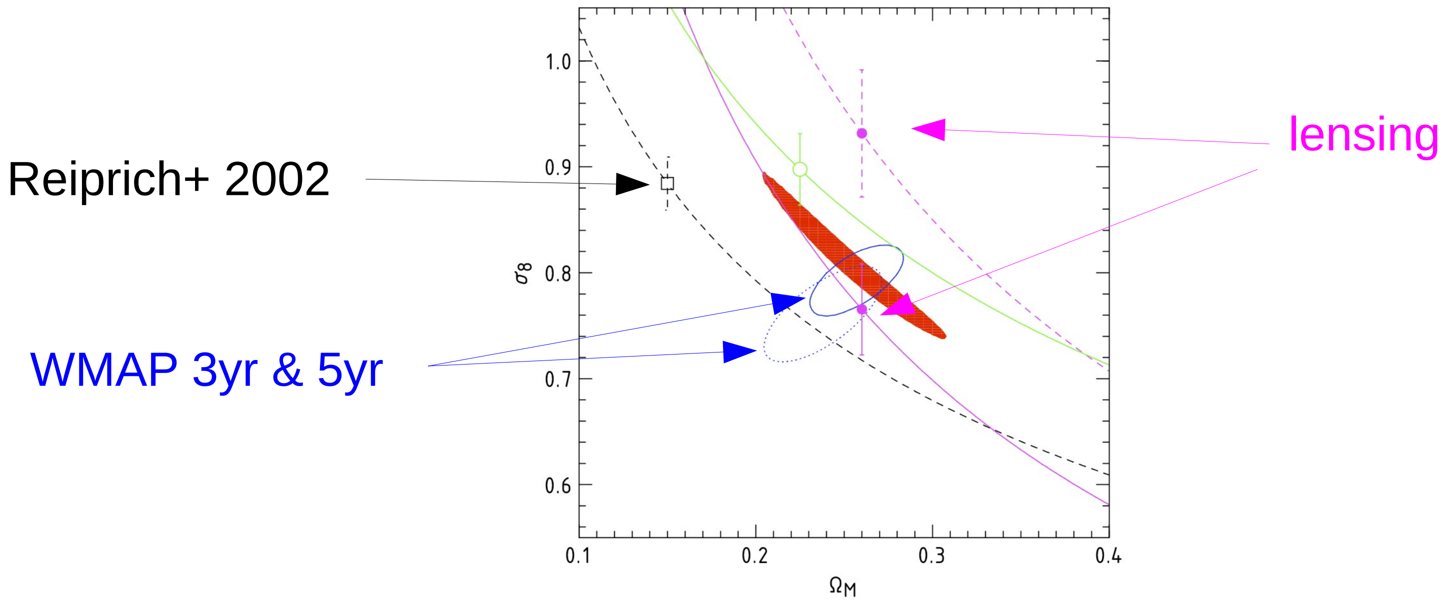


- ★ Note how **predicted** function and **measured** values are **both** sensitive to cosmology for high-z clusters

Experimental Results

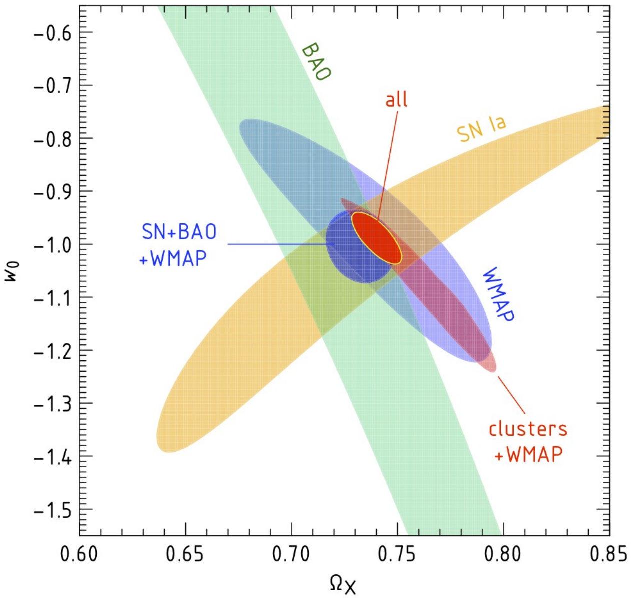
Vikhlinin's constraints on Ω_M and σ_8



- ★ different techniques give range in $\sigma_8 - \Omega_M$ plane
- ★ this work: $\sigma_8 = 0.813 \pm 0.012$
- ★ simulations depend on σ_8 - like higher values as get more clusters!

Experimental Results

Vikhlinin's constraints on Ω_Λ and w



combined constraints:

$$w = -0.991 \pm 0.045$$

$$\Omega_\Lambda = 0.740 \pm 0.012$$

- ★ assumed flat Universe here
- ★ note improvement of adding clusters
- ★ from cluster mass function alone: w = -1.14 +/- 0.21
  - ▸ recall cluster $f_{gas}(z)$: w = -1.14 +/- 0.31

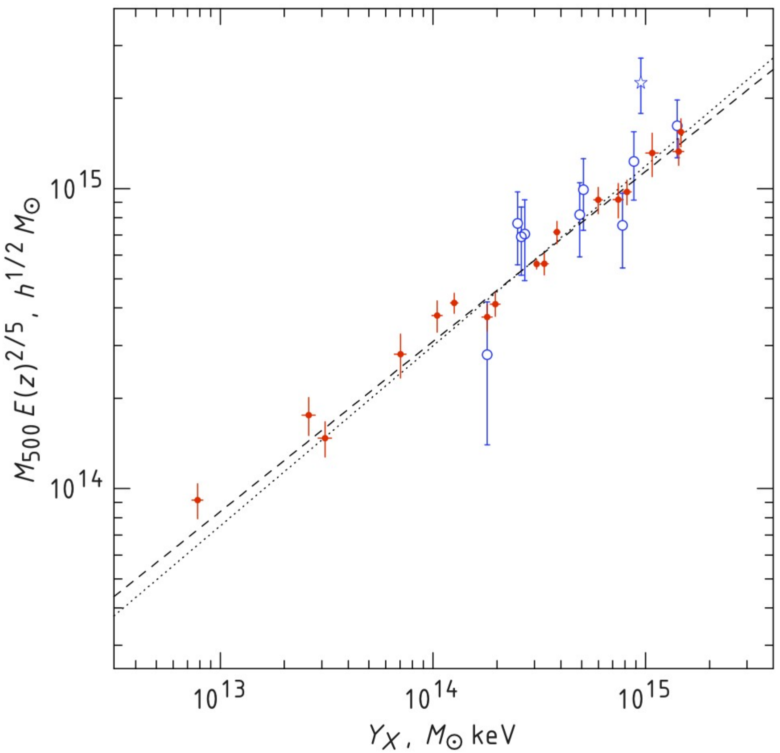
Caution: Mass Accuracy

Dominant source of error is mass scaling relations

- ⋆ LM relation for volume calculations
- ⋆ YM relation (or MT etc) for mass estimates

Vikhlinin et al (2009; ApJ, 692) tested X-ray YM relation against weak lensing masses

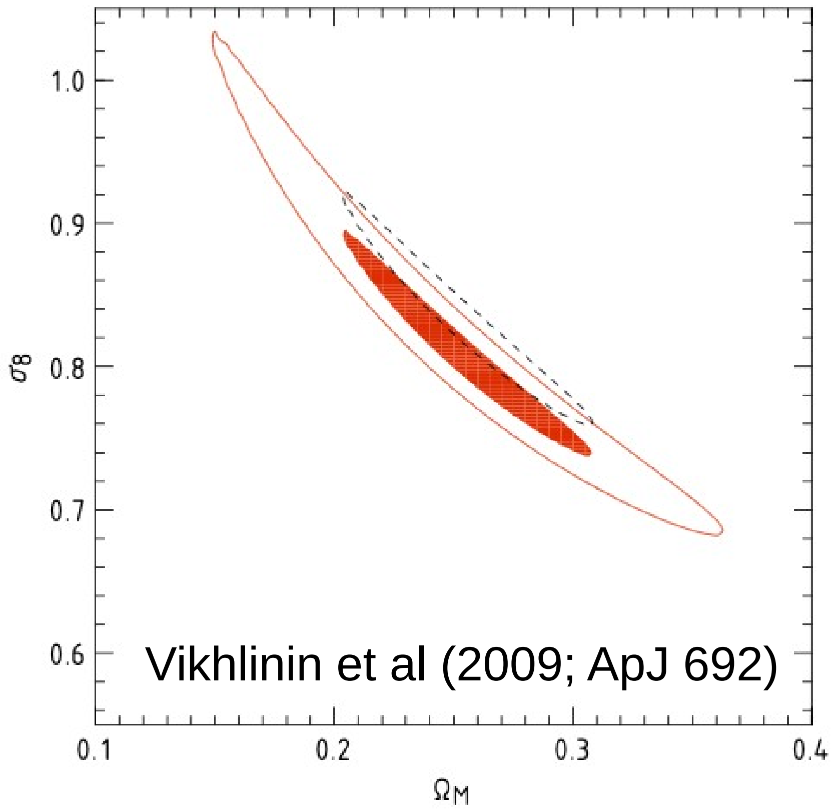
- ⋆ good agreement, but more precision required
- ⋆ tests of **evolution** of mass scaling relations needed



Caution: Mass Accuracy

Recall ~10% underestimate of X-ray masses c.f. simulations

- ⋆ black contour shows effect on σ_8, Ω_M



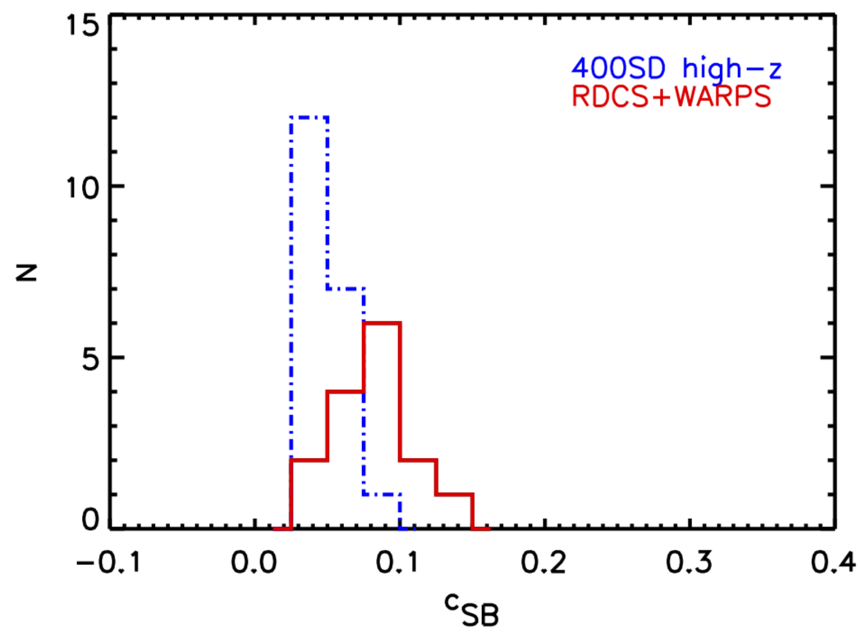
Caution: Selection Function

How well are selection functions known?

- ⋆ Santos et al (2010; A&A) compared surface brightness concentration c_{SB} for different high-z X-ray samples

Found significant difference in distributions

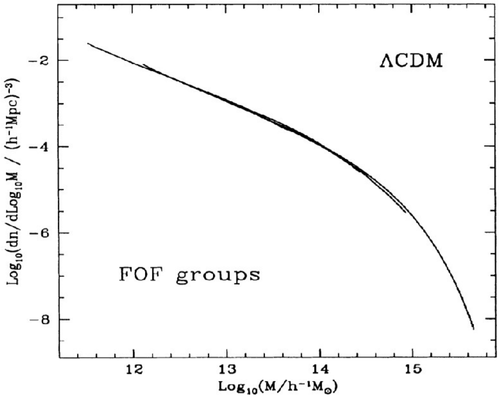
- ⋆ 400SD (Vikhlinin) survey missing concentrated clusters at high-z
- ⋆ clusters misclassified as point sources?
- ⋆ errors in selection function?



Caution: Selection Biases

High mass clusters are

- ★ brighter & rarer than low mass
- ★ z_{max} larger for high mass clusters
  - ▸ survey volume **much larger**



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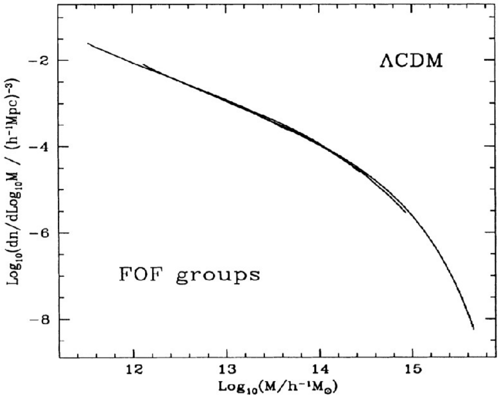
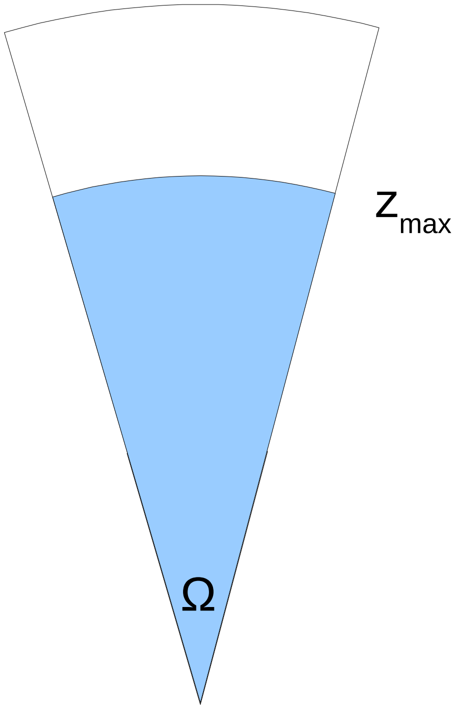


Few low mass clusters detected at high-z as too faint

Few high mass clusters detected at low-z as too rare (small volume)

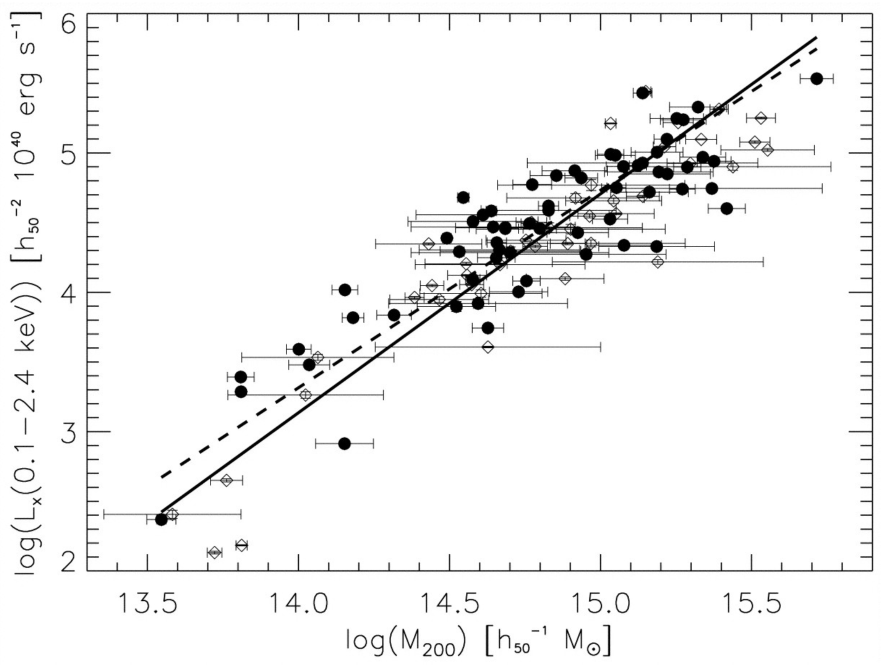
- ★ Mean z of massive clusters higher than low-mass clusters
- ★ **Malmquist bias** – accounted for by selection function

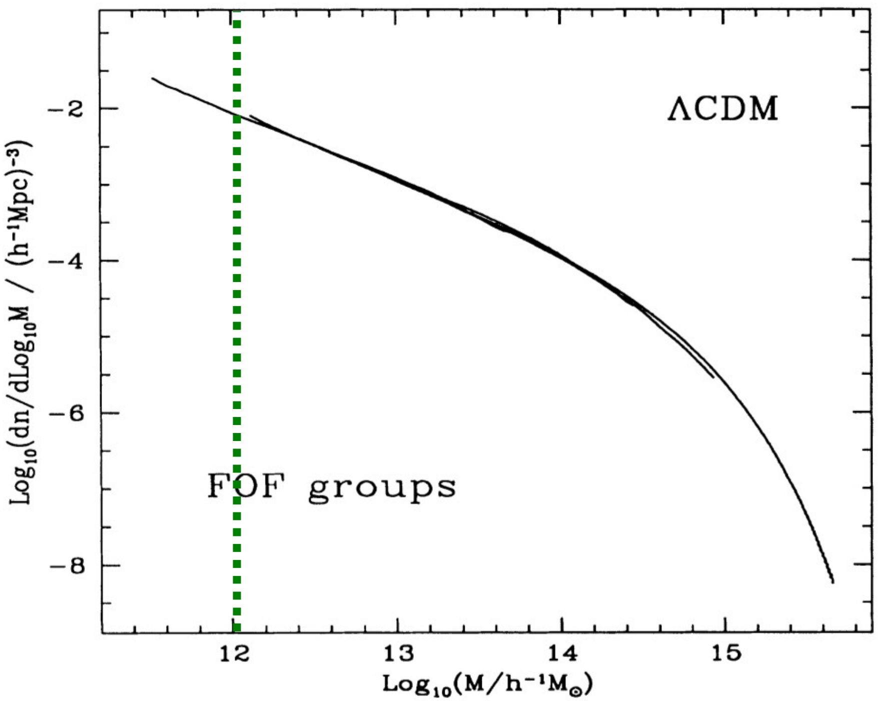


Caution: Selection Biases

Consider flux limited sample at some z

- ⋆ flux limit corresponds to some mass from LM relation
- ⋆ Scatter in L(M) means some clusters with masses too low will be in sample and vice-versa





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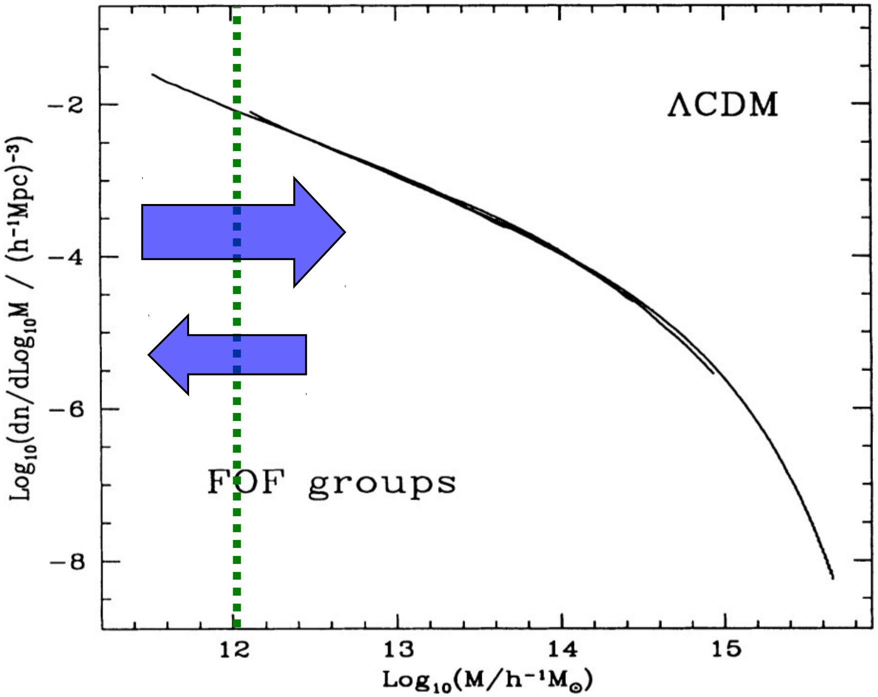
Slope of mass func means **more** clusters scattered **into** sample

- ★ Biases sample to clusters with Lx high for their M



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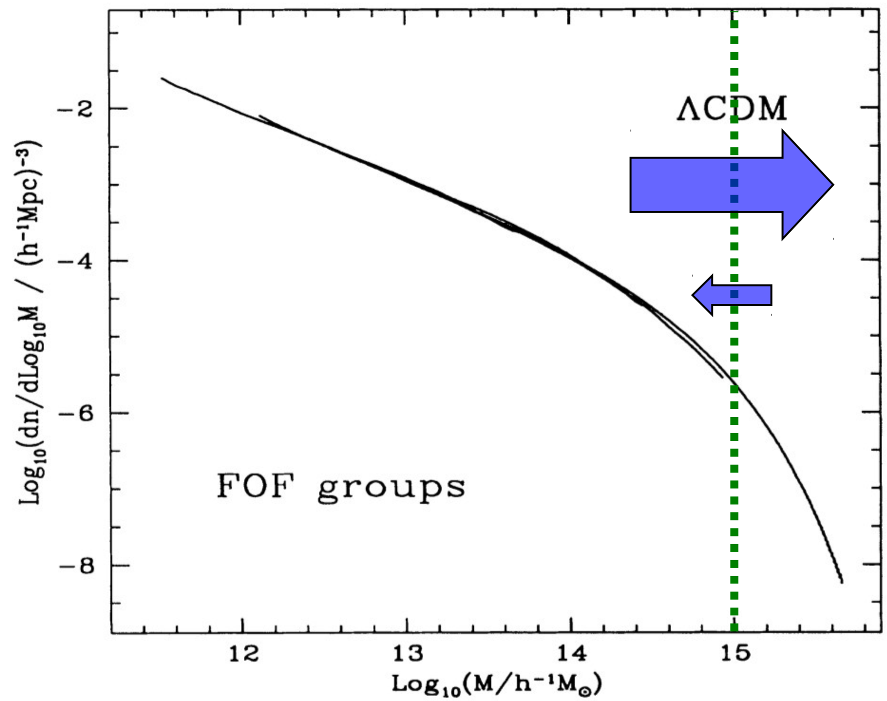
Slope of mass func means **more** clusters scattered **into** sample

- Biases sample to clusters with Lx high for their M
- Amount of bias depends on slope at limit & scatter



Caution: Selection Biases

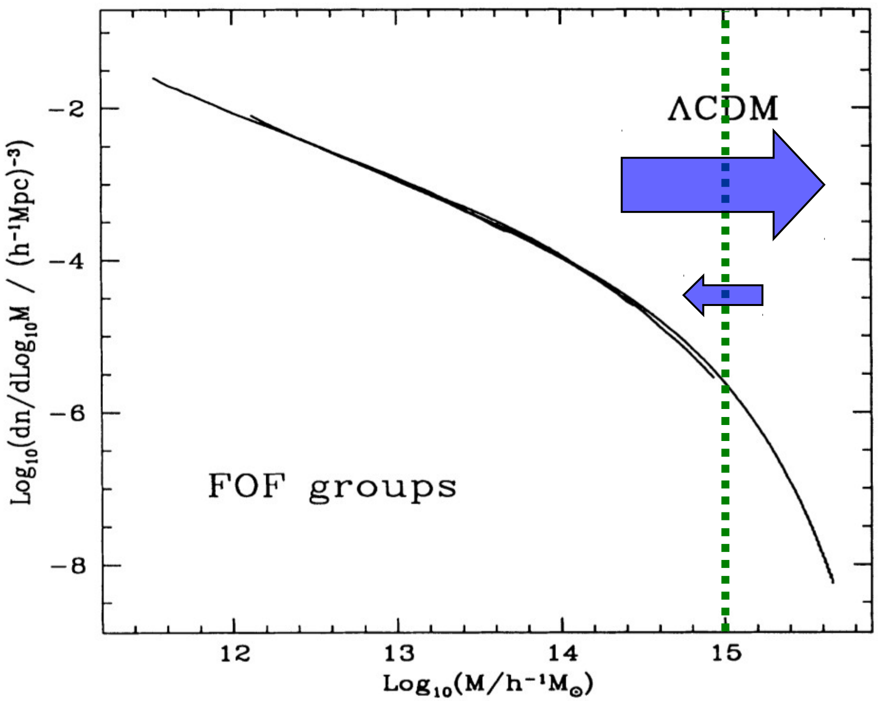
Known as **Eddington Bias**

★ overestimate cluster masses and number densities

Can correct if scatter in LM is known

★ does scatter vary with z?

★ hard to measure, but may decrease with z (Maughan 2007; ApJ, 668)



Summary 1

Cluster mass function sensitive to cosmology through

- ⋆ growth of structure – N(M)
- ⋆ geometry – d(z), V(z)

Large, well-calibrated X-ray samples measure shape and evolution of MF

- ⋆ selection function gives V(M,L)
- ⋆ best constraints from reobserving clusters to get T, Yx

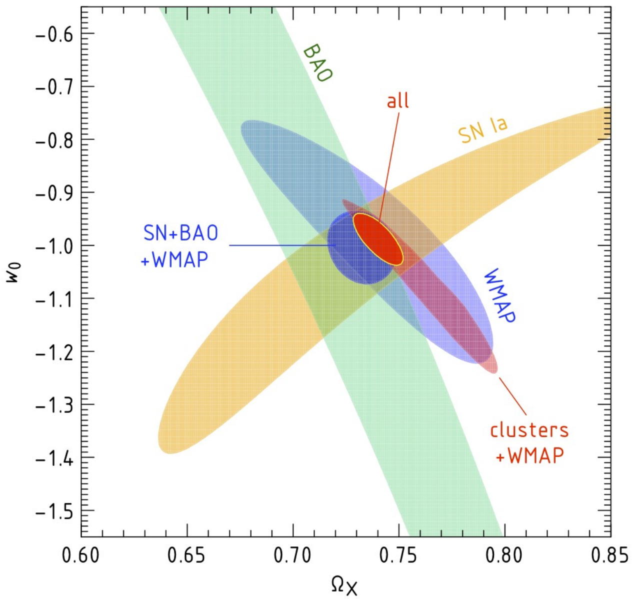
Mass uncertainties dominant source of error

- ⋆ affect M and V calculations

Selection function essential to control biases

Summary 2

Clusters powerful cosmological probes, with different sensitivities, assumptions to other methods



combined constraints:

$$w = -0.991 \pm 0.045$$

$$\Omega_\Lambda = 0.740 \pm 0.012$$

Current best bet:

★ flat Universe, 70% dark energy

★ DE is in form of cosmological constant (w=-1)