Density Parameter and Curvature

The density parameter is
\[ \Omega_0 = \frac{\rho_0}{\rho_c} \] [2.8]

Four possibilities:
(i) \( \Omega_0 = 0 \) : empty universe; constant expansion rate; (non-physical)
(ii) \( 0 < \Omega_0 < 1 \) : expansion overcomes gravity; universe expands forever
(iii) \( \Omega_0 = 1 \) : critical density; mass is just enough to stop it expanding but not enough to make it recollapse
(iv) \( \Omega_0 > 1 \) : gravity overcomes expansion; universe recollapses

Figure 2.4 Scale factor for different density parameters.

Age of the universe depends on the density parameter:
(i) \( \Omega_0 = 0 \), then \( \tau_0 = \tau_H \)
(ii) \( 0 < \Omega_0 < 1 \), then \( \frac{4}{3} \tau_H < \tau_0 < \tau_H \)
(iii) \( \Omega_0 = 1 \), then \( \tau_0 = \frac{2}{3} \tau_H \)
(iv) \( \Omega_0 > 1 \), then \( \tau_0 < \frac{4}{3} \tau_H \)

General relativity: mass density determines the curvature \( k \), or “shape”, of the universe:
(i) \( \Omega_0 = 1 \) : flat universe, \( k = 0 \)
(ii) \( \Omega_0 > 1 \) : closed (or bound) universe, \( k = +1 \)
(iii) \( \Omega_0 < 1 \) : open (or unbound) universe, \( k = -1 \)
Curvature affects the geometry of the universe:
(i) parallel light rays do not stay parallel
(ii) flux from a distant galaxy, \( F \neq \frac{L}{4\pi A} \)
(iii) angular size of a distant radio source, \( \theta \neq \frac{D}{d} \)

Measuring the Density Parameter

mass density = mass of galaxy \( \times \) number density

Luminous matter

\( M_{gal} = \text{number of stars} \times \text{mass of star} \)
\[ \Rightarrow \quad \Omega_0 \sim 0.002 \]

Orbital velocities

Circular motion

\[ \frac{mv^2}{r} = \frac{GM_{gal} m}{r^2} \]

\[ M_{gal} = \frac{rv^2}{G} \]

Spiral galaxies: flat rotation curves imply galaxies are embedded in a halo of dark matter
\[ \Rightarrow \quad \Omega_0 \sim 0.03 \]
Virial theorem

Gravitational system in equilibrium

\[ 2 KE + PE = 0 \]

\[ M = \frac{3 r \sigma_r^2}{f G} \]

where \( \sigma_r \) is the radial velocity dispersion, and \( f \sim 1 \).

Elliptical galaxies: \( M_{gal} \sim 30 \times \) luminous matter, implying a dark matter halo

\[ \Rightarrow \quad \Omega_0 \sim 0.03 \]

Galaxy clusters: \( \sigma_r \) is from galaxy redshifts; virial mass \( \gg \) mass of the galaxies

\[ \Rightarrow \quad \Omega_0 \sim 0.15-0.3 \]

X-ray cluster gas

Gas mass is found from the surface brightness profile of x-ray emission

\[ \Rightarrow \quad \Omega_0 \sim 0.3 \]

Gravitational lensing

Galaxy clusters deflect the light paths of sources behind them like a giant magnifying glass. The amount of deflection measures the mass of the cluster.

\[ \Rightarrow \quad \Omega_0 \sim 0.3 \]
Figure 2.7  *Chandra* X-ray image of the cluster around 3C295. (NASA/CXC/SAO)

**Dark Matter**

Luminous matter is $\sim 1\%$ of total mass  
Baryonic matter is $\sim 15\%$ of total mass  

$\Rightarrow$  85% of universe is non-baryonic dark matter

Candidates:  
• “massive” neutrinos – oscillations of solar neutrinos give $0.05 < m_{\nu_e} < 3 \text{ eV}$ or $0.001 < \Omega_0 < 0.18$ for all $\nu$  
• cold dark matter (CDM) – exotic elementary particles with $m \geq 10 \text{ GeV}$, only interact via gravity (today), $\Omega_0 = \, ??$
Figure 2.8 Gravitational lensing in Abell 2218. (©W.Couch, R. Ellis, and NASA; see http://hubble.stsci.edu/newscenter/archive/1995/14/)
Cosmological Constant

In GR, the vacuum can have a potential energy:

\[ E_\Lambda \propto -\Lambda r^2 \]

where \( \Lambda \) is the cosmological constant.

Acts like a pressure or “anti-gravity” – causing the expansion to accelerate.

The universe can be older than \( \tau_H \).

**Figure 2.9** Scale factor for \( \Lambda \)-dominated flat universe.

Vacuum density parameter

\[ \Omega_\Lambda = \frac{\Lambda}{3H_0^2} \]

Any universe with \( \Omega_0 + \Omega_\Lambda = 1 \) has a flat geometry.

Recent observations of the CMB & high-redshift supernovae suggest that the universe is actually accelerating: \( \Omega_0 \approx 0.3, \Omega_\Lambda \approx 0.7 \).
Summary

Density parameter:
\[ \Omega_0 = \frac{\rho_0}{\rho_c} \]

Curvature:
(i) open, \( 0 < \Omega_0 < 1 \)
(ii) critical, \( \Omega_0 = 1 \)
(iii) closed, \( \Omega_0 > 1 \)

Measurements of the density parameter:
- luminous matter, \( \Omega_0 \sim 0.002 \)
- spiral galaxy rotation curves, \( \Omega_0 \sim 0.03 \)
- elliptical galaxies, virial theorem, \( \Omega_0 \sim 0.03 \)
- clusters, virial theorem, \( \Omega_0 \sim 0.15-0.3 \)
- clusters, x-ray gas, \( \Omega_0 \sim 0.3 \)
- clusters, gravitational lensing, \( \Omega_0 \sim 0.3 \)

\[ \Rightarrow \] 85% of universe is non-baryonic dark matter

Cosmological constant: vacuum has a pressure ("anti-gravity").
Find \( \Omega_0 \simeq 0.3, \Omega_\Lambda \simeq 0.7 \)