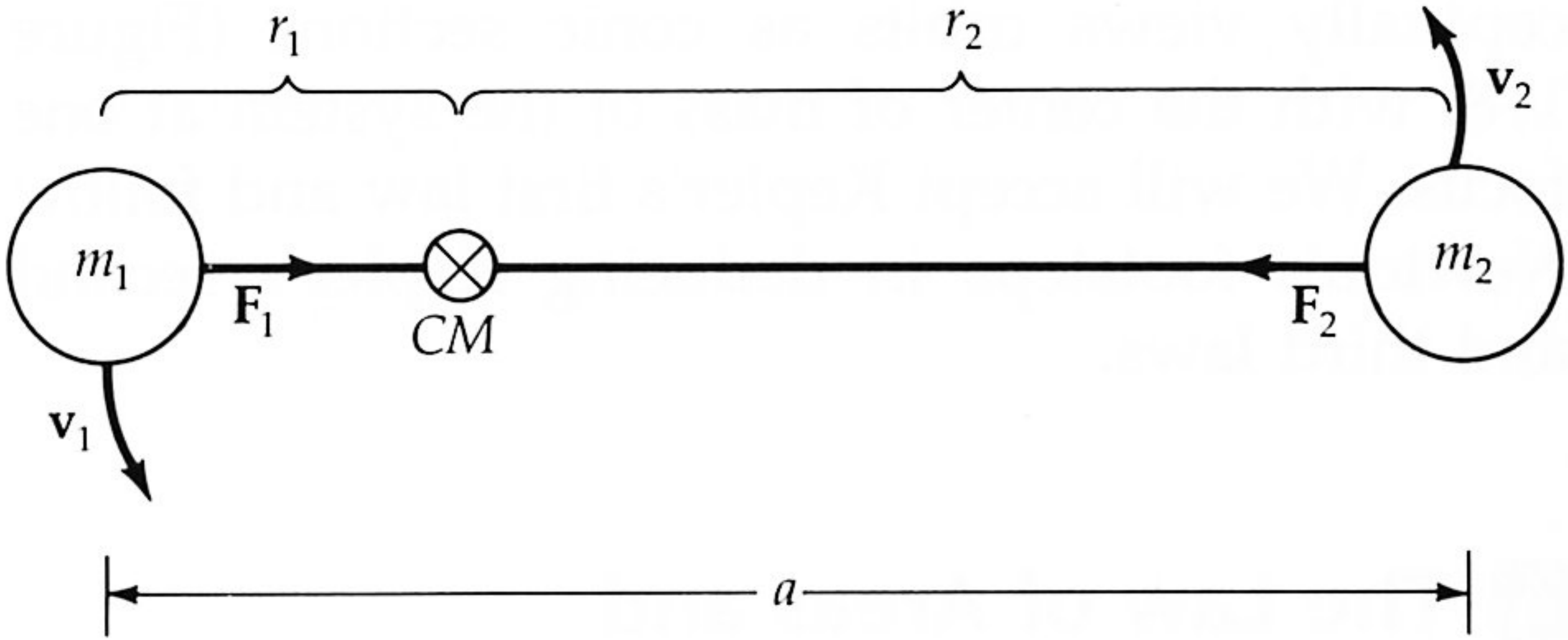
Stellar Masses

- Binary systems
- Kepler's 3rd Law
- Visual binaries
- Spectroscopic binaries

Binary Systems

- consider two stars with masses M_1 and M_2 in circular orbits around their centre of mass (CM)
- radius of each orbit is r_1 and r_2 respectively and the total separation is a
- can use Newton’s Laws and circular motion to determine masses



Zeilik Fig 1-14

Circular Motion

$$F_1 = \frac{M_1 v_1^2}{r_1} = \frac{4\pi^2 M_1 r_1}{P^2}$$

and

$$F_2 = \frac{M_2 v_2^2}{r_2} = \frac{4\pi^2 M_2 r_2}{P^2}$$

where P is the period which is the same for both stars

Centre of Mass

- definition of centre of mass means

$$M_1 r_1 = M_2 r_2$$

Newton's Law of Gravity

$$F_1 = F_2 = \frac{GM_1M_2}{a^2}$$

where

$$a = r_1 + r_2$$

Newton's form of Kepler's Third Law

- combining these three equations gives

$$\frac{4\pi^2 M_1 r_1}{P^2} = \frac{G M_1 M_2}{a^2}$$

$$P^2 = \frac{4\pi^2 a^2 r_1}{G M_2}$$

Eliminate r_1 using

$$a = r_1 + r_2 = r_1 + \frac{M_1}{M_2} r_1 = \left(\frac{M_1 + M_2}{M_2} \right) r_1$$

so

$$P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)}$$

and

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2}$$

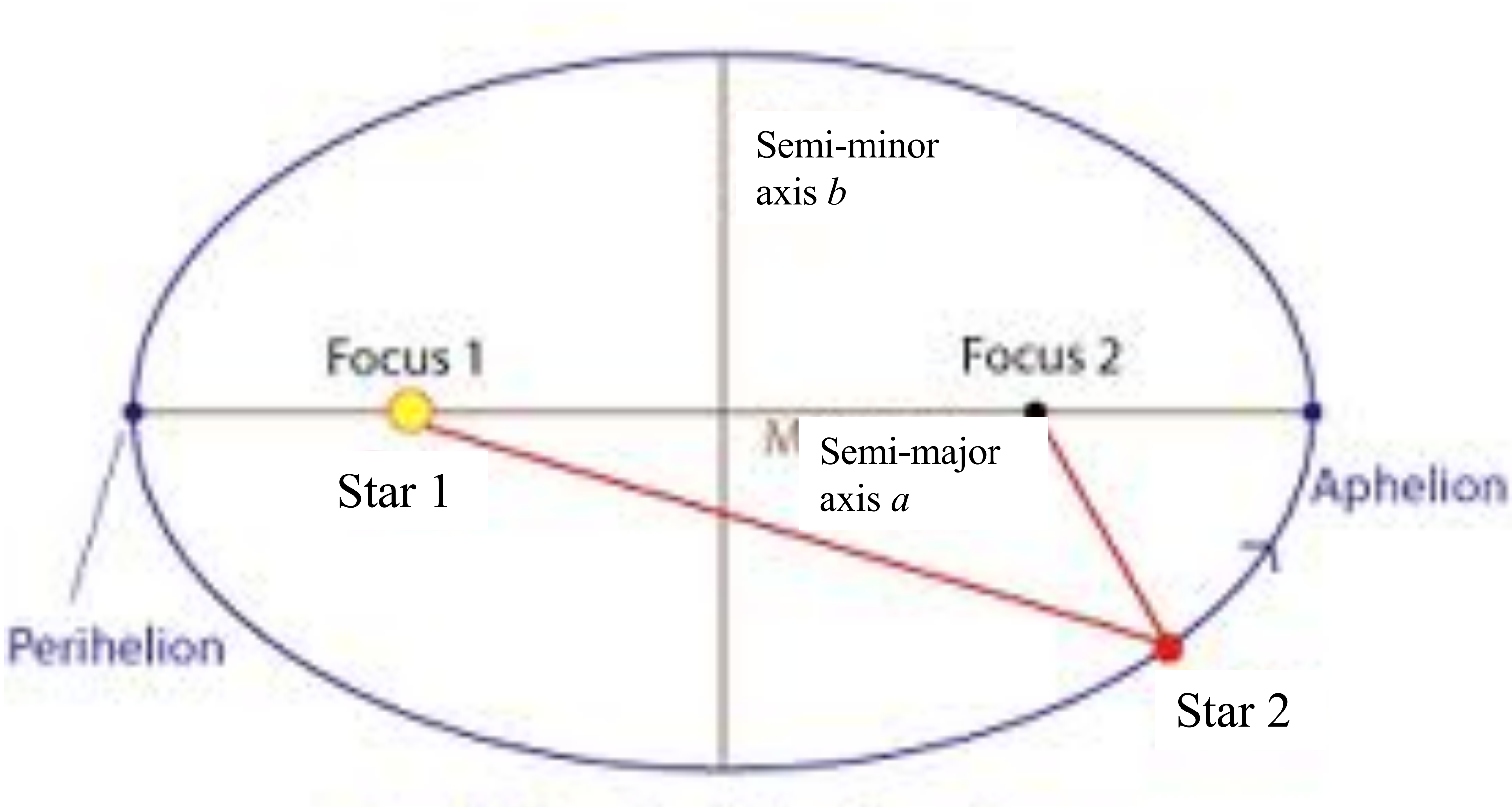
Real Orbits

- orbits are generally elliptical and described by their semi-major axis a and semi-minor axis b
- eccentricity is defined by

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

i.e. $e = 0 \quad \Rightarrow$ circular orbit

- Newton's form of Kepler's third law also applies to elliptical orbits



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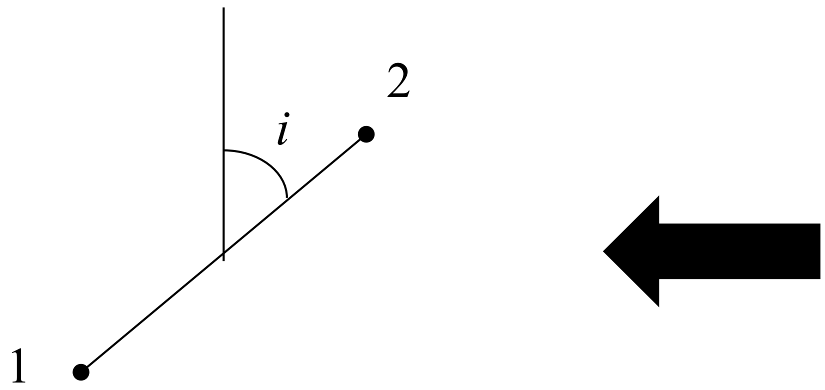
Orbital Inclination

- in general the orbital plane of a binary system will be inclined by some angle i to the plane of the sky:

$i = 0^o \quad \Rightarrow$ face on

$i = 90^o \quad \Rightarrow$ edge on

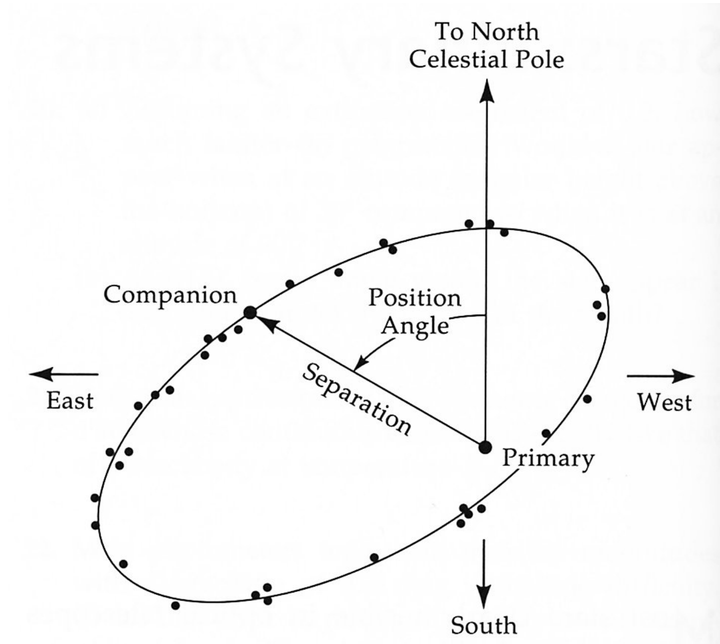


Types of Binary System

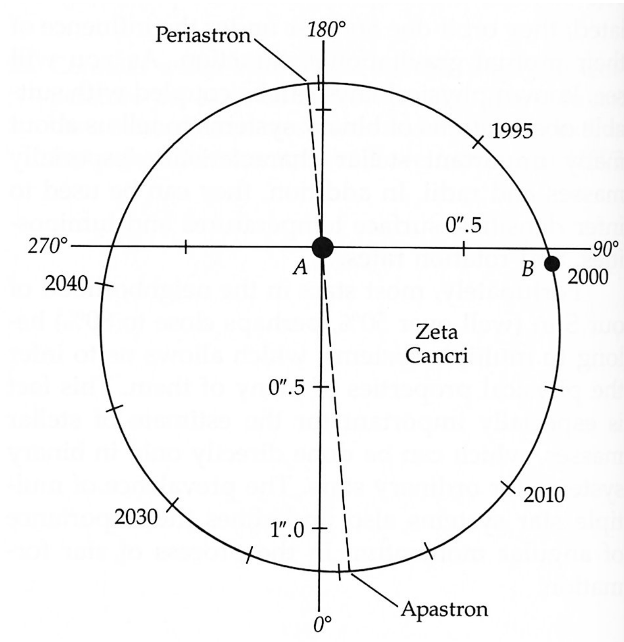
- Visual binaries
  - Two stars spatially resolved on the sky in orbit around each other
- Spectroscopic binaries
  - Two stars not spatially resolved, but orbital motion revealed through periodic Doppler shifts of their spectral lines

Visual Binaries

- Can measure sum of masses from Kepler's law and ratio of masses from ratio of semi-major axes and hence can solve for individual masses



(A)



(B)

Masses from Spectroscopic Binaries

- for circular orbits the orbital velocities are

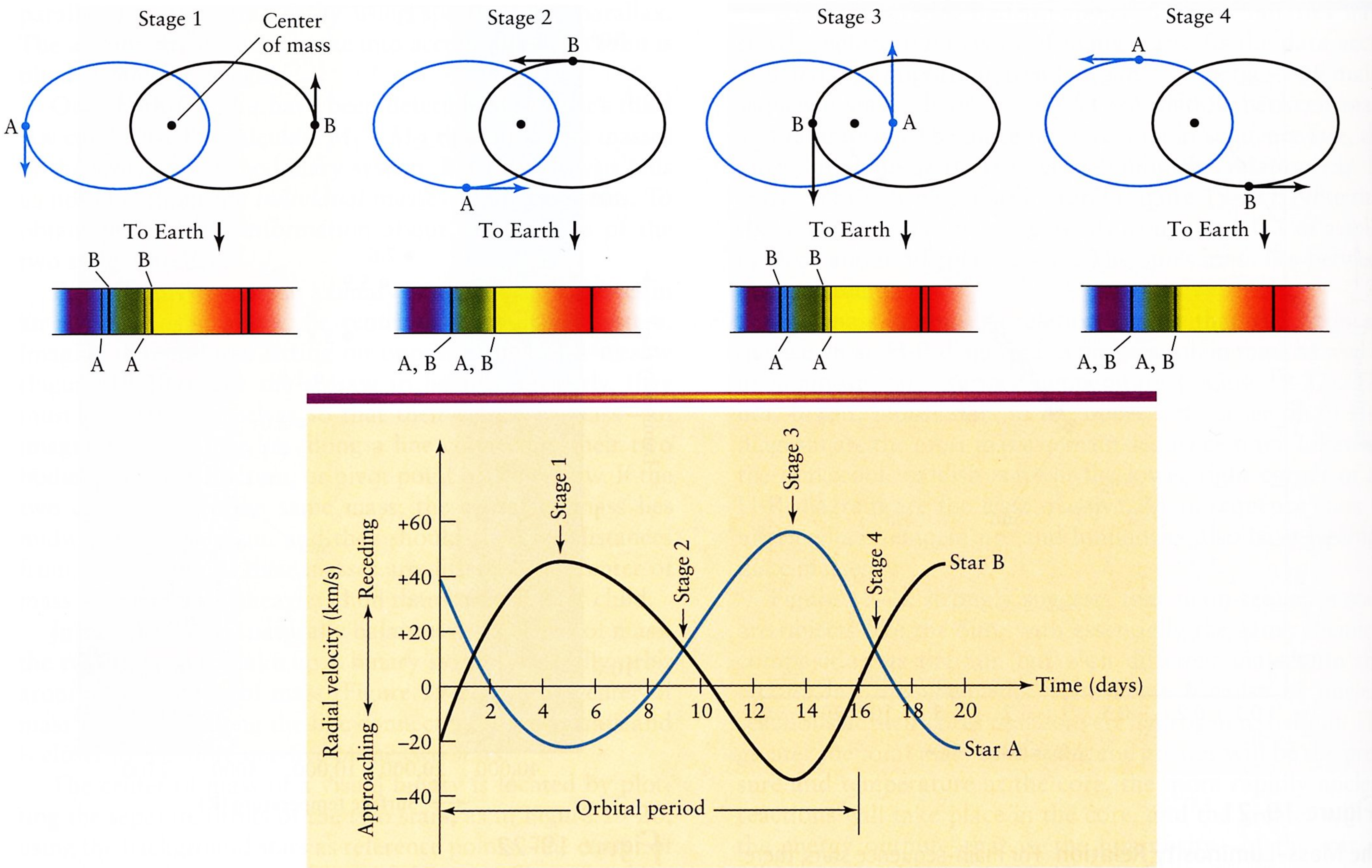
$$v_1 = \frac{2\pi r_1}{P} \quad \text{and} \quad v_2 = \frac{2\pi r_2}{P}$$

- for inclination angle i the observed radial velocities are

$$v_{r1} = v_1 \sin i \quad \text{and} \quad v_{r2} = v_2 \sin i$$

- If we see lines from both stars can determine mass ratio from

$$\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$


Stage 1
Center of mass
A
B
To Earth
B B
A A
Stage 2
B
A
To Earth
A, B A, B
Stage 3
B
A
To Earth
B B
A A
Stage 4
A
B
To Earth
A, B A, B
Radial velocity (km/s)
Approaching
Receding
+60
+40
+20
0
−20
−40
Stage 1
Stage 2
Stage 3
Stage 4
Star B
Star A
Time (days)
2 4 6 8 10 12 14 16 18 20
Orbital period

Also

$$a = r_1 + r_2 = \frac{P}{2\pi}(v_1 + v_2) = \frac{P}{2\pi}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)$$

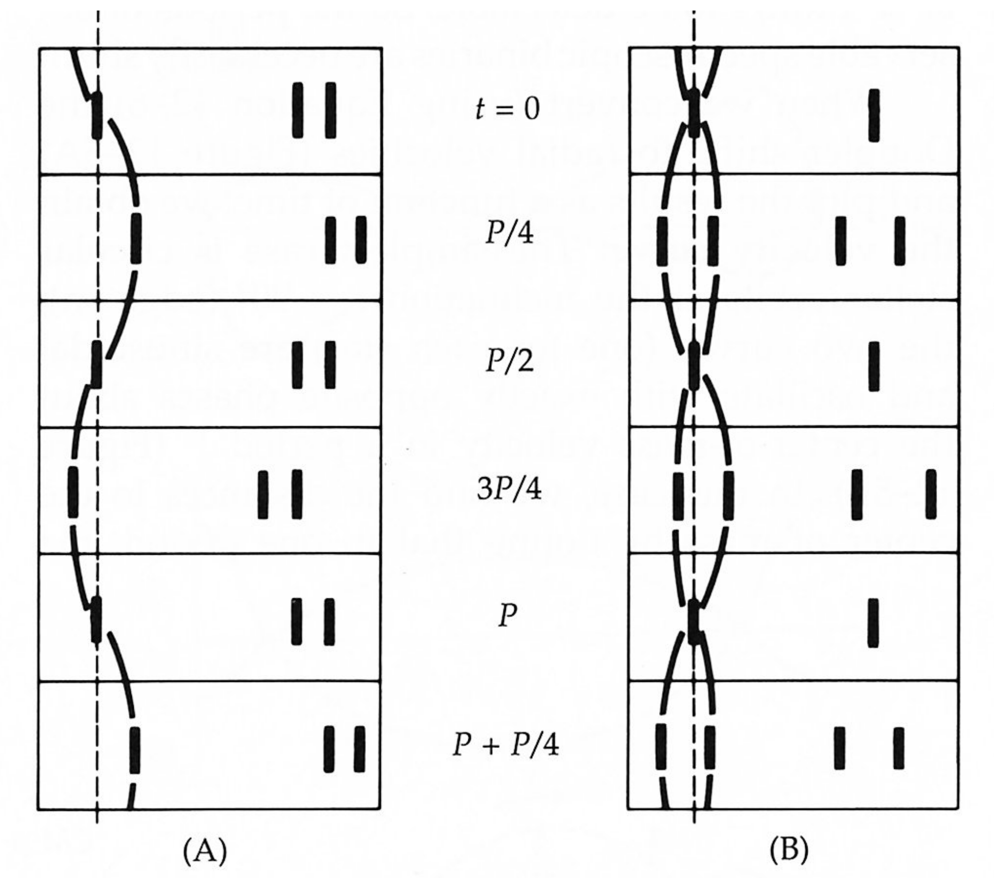
so from Kepler's law

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} = \frac{P}{2\pi G}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

i.e. only a lower limit to the sum of the masses

Single-lined Spectroscopic Binaries

- only one spectrum is observed say v_{r1}



Zeilik Fig 12-4

- so eliminate v_{r2}

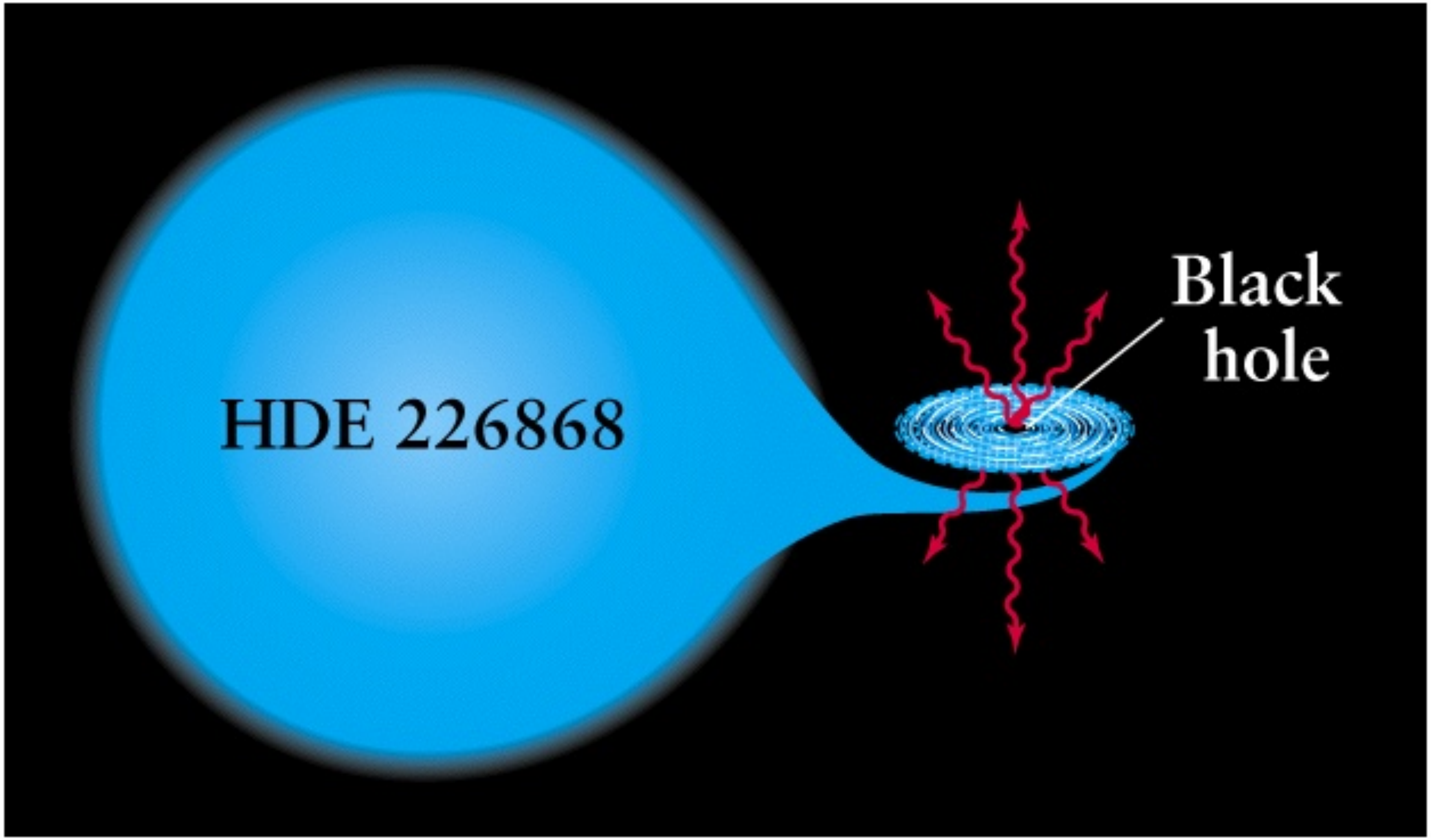
$$M_1 + M_2 = \frac{P}{2\pi G}\left(\frac{v_{r1} + \frac{M_1}{M_2} v_{r1}}{\sin i}\right)^3$$

$$M_1 + M_2 = \frac{P v_{r1}^3}{2\pi G}\left(\frac{\frac{M_1 + M_2}{M_2}}{\sin i}\right)^3$$

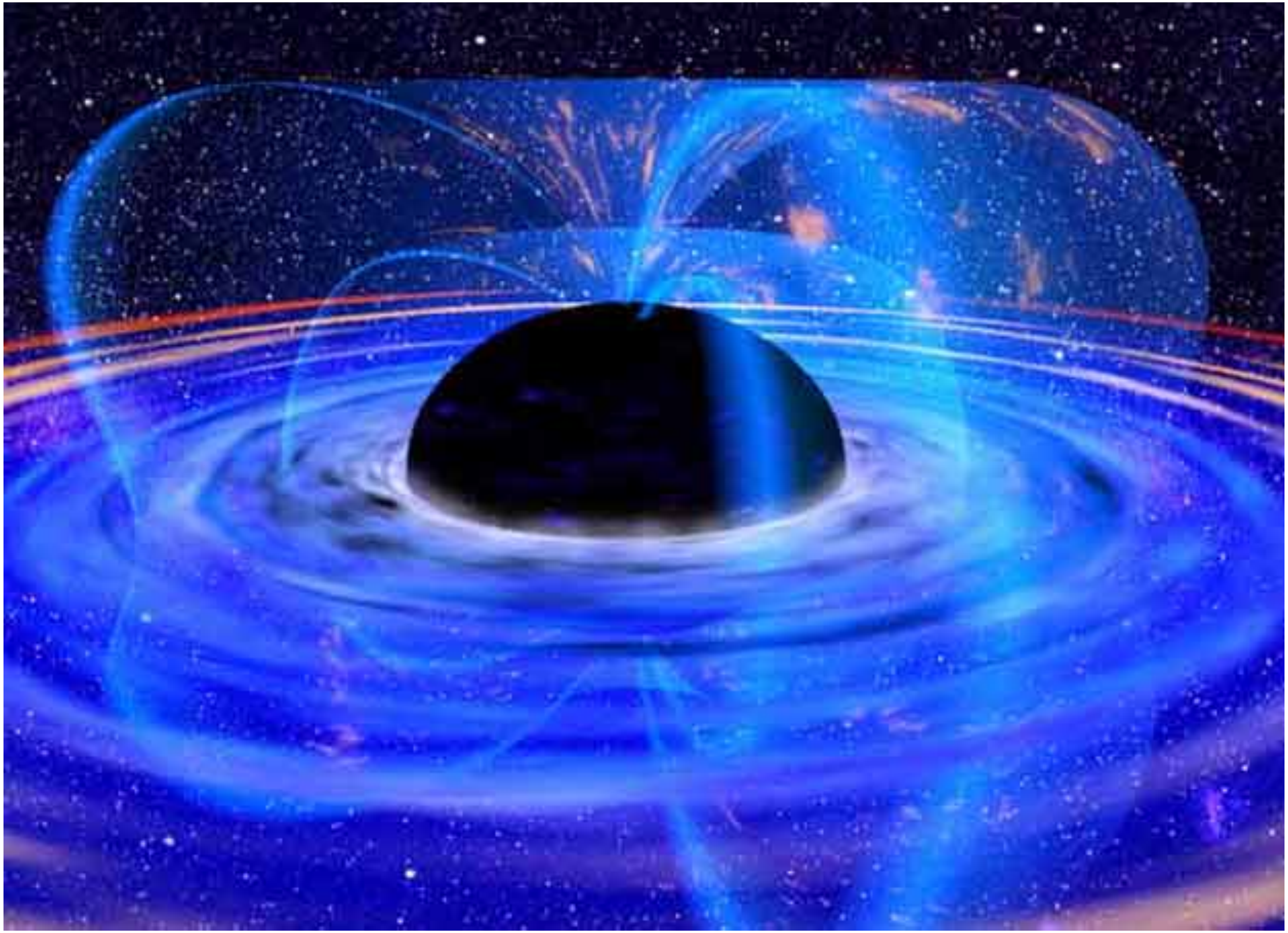
so $$\frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{P v_{r1}^3}{2\pi G}$$

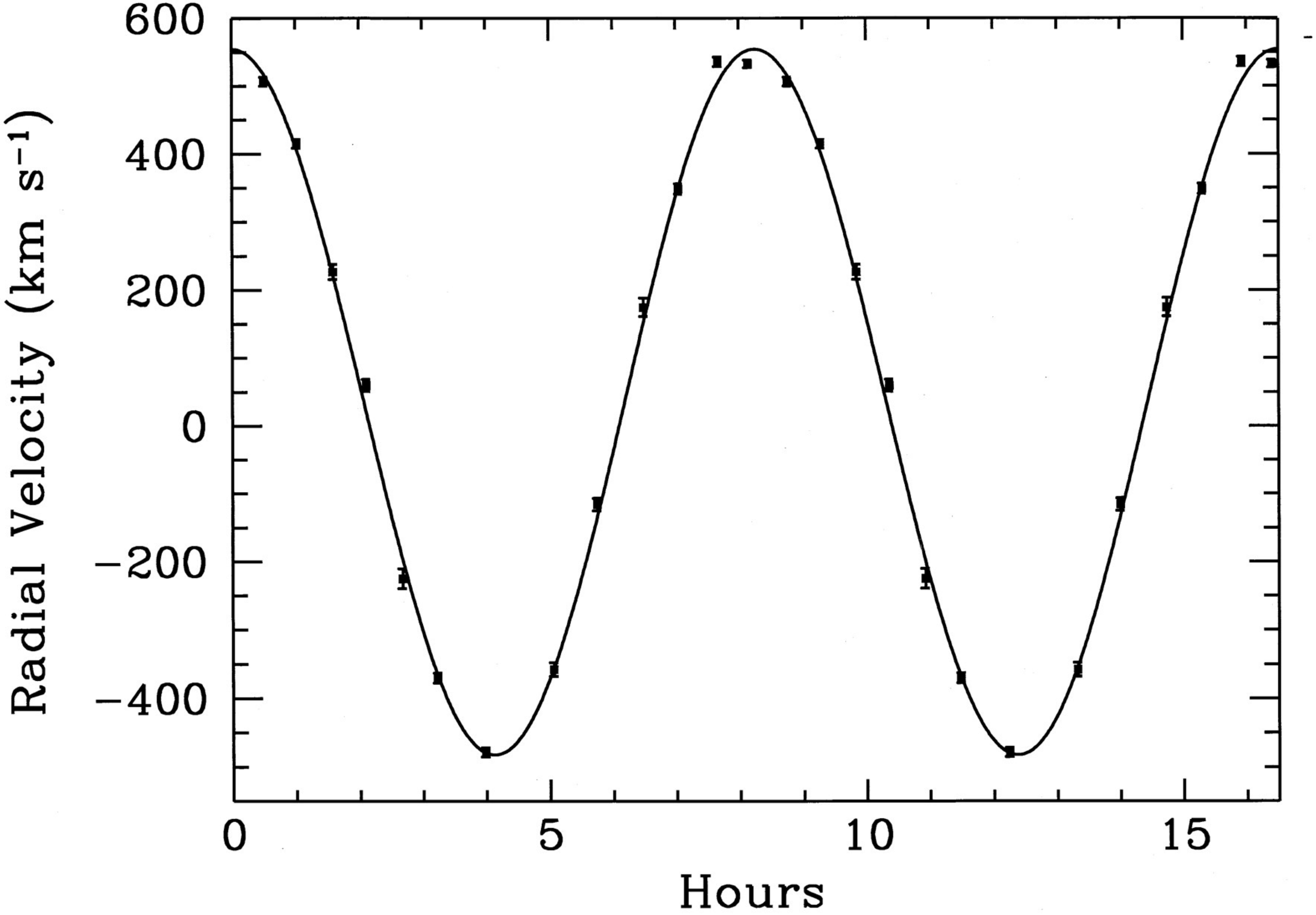
i.e. if we can estimate M_1

we can constrain M_2



a





Radial-velocity curve of the visible star in the X-ray binary GS 2000 + 25
Fillipenko et al. (1999) www.pnas.org/content/96/18/9993.full
Shows that invisible compact companion star is a 5 solar mass black hole

Summary

- visual binaries provide accurate masses, but not many known
- spectroscopic binaries only usually constrain the masses with inclination the greatest uncertainty unless the system is eclipsing
- spectroscopic binaries used to find black holes and planets orbiting other stars